Assessing Predictability with a Local Ensemble Kalman Filter

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Abstract

In this paper the spatio-temporally changing nature of predictability is studied in the National Centers for Environmental Prediction (NCEP) Global Forecast System (GFS), a state-of-the-art numerical weather prediction model. Atmospheric predictability is assessed in the perfect-model scenario for which forecast uncertainties are entirely due to uncertainties in the estimates of the initial states. Initial conditions (analyses) are obtained by assimilating simulated noisy observations of the “true” states with the Local Ensemble Kalman Filter (LEKF) data assimilation scheme.

For this specific choice of the model and data assimilation system, the forecast errors grow exponentially in the extra-tropics and linearly in the tropics. The analysis errors are the smallest in the regions, the extratropical storm tracks, where the growth of the forecast errors is the fastest. This seemingly paradoxical result is due to the strong anti-correlation between the local dimensionality and the error variance explained by the LEKF ensemble. This strong anti-correlation makes the LEKF algorithm extremely efficient in estimating the analysis and forecast uncertainties in the regions of local low dimensionality, which coincide with the regions fastest error growth. The efficient estimation of the space of uncertainties enables the LEKF to produce very accurate analyses and very accurate estimates of the forecast uncertainties. It is conjectured that the results presented here could be reproduced with other suitably formulated ensemble-based Kalman filter data assimilation schemes.
1 Introduction

In dynamical systems theory, predictability is often characterized by the largest Lyapunov exponent of the system. This characterization is based on studying the evolution of initially small perturbations to a nonlinear trajectory, assuming that a numerically computed, sufficiently long trajectory can explore the small neighborhood of all possible states of the system (e.g., Ott 2003). Such a characterization may not apply for finite time forecasts and is especially inappropriate when the dimensionality of the dynamics is so high that exploration of the attractor by a typical trajectory takes a very long time. This is the case for a high-dimensional weather prediction model that mimics the evolution of the atmosphere. In such cases, a crucial issue is the proper selection of initial conditions and perturbations thereto that are used for the assessment of predictability.

We carry out experiments for the perfect model scenario: a “true” nonlinear trajectory is generated by a long integration of the model from a realistic Northern Hemisphere winter initial condition. Then, imperfect (perturbed) initial conditions are obtained by assimilating simulated noisy observations of the “true” states with the local ensemble Kalman filter (LEKF; Ott et al. 2004) data assimilation system. We take advantage of our previous work to test an implementation of the LEKF on the NCEP GFS (Szunyogh et al. 2005; SEA05 hereafter): an important feature of the previous work is that the observations are randomly distributed. Thus, unlike a real observing network, the simulated observing network may be assumed to have no effect on the geographical distribution of the analysis and forecast uncertainties (provided that the observational network is not too sparse). The analyses and analysis errors were studied in detail in SEA05. Here the focus is on the spatio-temporal evolution of the forecasts and forecast uncertainties started from the analyses of SEA05. In this sense, the present paper can be viewed as a sequel to SEA05.

In addition to the analyses, the LEKF also provides an ensemble of initial perturbations that represents the uncertainty in the initial conditions. We utilize this information to generate forecast ensembles and to calculate the ensemble dimension (E-dimension; Patil et al.
2001; Oczkowski et al. 2005), a measure of the spatio-temporally changing complexity of the forecast uncertainties. One motivation for carrying out these calculations is the strong anti-correlation that we found in SEA05 between the E-dimension and the efficiency of the ensemble in capturing the true analysis uncertainties. Another motivation is the close relationship that we have observed between local atmospheric instabilities and local regions of low E-dimensionality for an ensemble of bred vectors in Oczkowski et al. (2005). These results suggest that, in the regions of most rapidly growing forecast errors, the forecast uncertainties may be predictable with exceptional accuracy. Indeed, the results presented here demonstrate that, in the perfect-model scenario, the LEKF best captures the forecast uncertainties in the regions of largest forecast errors.

The analysis-forecast system used in our experiments, as well as the experimental design, are briefly described in Section 2. Section 3 investigates the geographical distribution and typical evolution of the forecast errors. This section also provides a detailed account of a case of explosive error growth. Section 4 investigates the relationship between the E-dimension, forecast error growth and the skill of the ensemble in tracking the space of the spatio-temporally evolving forecast uncertainties. Section 5 is a summary of our main conclusions.

2 Experimental design

The LEKF scheme is a model-independent algorithm to estimate the state of a large spatio-temporally chaotic system (Ott et al. 2004). The term “local” refers to an important feature of the scheme: it solves the Kalman filter equations locally in model grid space. More precisely, the state estimate at a grid point \( P \) is obtained independently from the state estimate at the other grid points, considering the observations and the background state only from a local cube centered at \( P \). The LEKF scheme also provides an estimate of the analysis uncertainty at \( P \) and generates an ensemble of analysis perturbations that provide a representation of the estimated uncertainty at \( P \). In what follows we employ a 4% multiplicative variance inflation (Anderson and Anderson 1999) at each analysis step to increase the estimated
analysis uncertainty in order to compensate for the loss of ensemble variance due to sampling errors and the effects of nonlinearities. The scheme then has two tunable parameters: the number of grid points in the local cube and the number of ensemble members.

In SEA05 the “true” state was generated by a 60-day integration of the model starting from the operational NCEP analysis at 0000 UTC on 1 January 2000. The two components of the horizontal wind vector and the temperature were observed at all model levels, and the associated surface pressure was also observed. The assumed observational errors were normally distributed with zero mean and 1 m/s, 1 K and 1 hPa standard deviation. Initially, observations were generated at all (17,848) horizontal grid point locations. Then, reduced observational networks were created by gradually removing observational locations at randomly selected grid points. This approach was applied to construct three additional observational networks taking vertical sounding of the atmosphere at 2000, 1000, or 500 fixed locations every six hours.

Based on the results of the numerical experiments, SEA05 drew the general conclusion that the LEKF scheme was a highly accurate data assimilation scheme in the perfect model scenario. SEA05 also found that the analysis errors formed characteristic geographical patterns:

- The largest time mean analysis errors occurred in the main regions of parameterized deep convection in the tropics. The same regions were also found to be regions of the highest E-dimension.

- The analysis errors had extremely small magnitude in the main extra-tropical storm track regions. These regions were also found to be regions of the lowest E-dimension.

In what follows, we investigate the subsequent evolution of the distribution of the forecast errors. Most of the results presented here are for a configuration of the LEKF that consists of a 40-member ensemble, $7 \times 7 \times v$ grid point local cubes ($v$ is the number of vertical grid points in the cube and changes with altitude; see SEA05 for details), and the number of simulated vertical soundings is 2000.
2.1 Data sets

A state estimate is obtained every six hours by assimilating the simulated observations with the LEKF scheme. Forecast trajectories are started from the 0000 UTC and 1200 UTC analyses each day. An ensemble of forecasts is also started every 12 hours, using the analysis ensemble provided by the LEKF as the initial conditions. Forecast error statistics are generated by comparing the forecasts to the true states. The forecast error statistics are computed for the 40-day period that starts at the fifteenth day along the “true” trajectory. We refer to time using the 40-day period as reference, i.e., the first forecast that we verify starts at 0000 UTC on day 1 (of the 40-day period), and the last forecast we verify starts at 1200 UTC on day 40. We present error statistics in the following formats:

- Snapshots of errors are presented by mapping the difference between the forecast and the true state on the grid.

- Maps of the sample mean errors are generated by first computing the absolute value of the difference between the forecasts and the true states at the grid points and then computing the sample mean of the absolute values.

- The error for a geographical region is obtained by computing the root-mean-square of the error over all grid points in the geographical region. This information is presented in plots showing time series of these errors.

- The sample mean error for a geographical region is obtained by computing the sample mean of the root-mean square error for the given geographical region.

3 Evolution of the forecast errors

The simulations in SEA05 found that the largest wind and temperature analysis errors were in the main regions of deep convection in the tropics, while the smallest analysis errors were found in the mid-latitude storm track regions. This initial distribution rapidly changes as
the forecasts progress. Figure 1 illustrates this rapid evolution, showing the sample mean of the forecast errors for the meridional component of the wind vector at the 500 hPa level (the figure shows the sample mean over all 80 forecast cycles). There seems to be a relationship between the errors in the region of deep convection and the early amplification of the errors in the north-Pacific storm track region. Then the errors propagate westward along the upper tropospheric wave guides depicted in Chang (1999a and 1999b). Although a clear indication of rapidly growing errors in the North-Atlantic and Southern Hemisphere storm track regions can be seen first at the 48-hour forecast lead time, the storm track regions become the location of the dominant error patterns in the extra-tropics by the 72-hour forecast lead time.

3.1 Dependence on the geographical region

The difference between the error growth mechanisms in the tropics and the extra-tropics becomes obvious by investigating the time evolution of the root-mean-square forecast errors for the different geographical regions (Figure 2). The most striking difference between the tropics and extra-tropics is in the functional dependence of the error growth on the forecast lead time. In the tropics, the root mean square of the forecast error, \( z_f(t) \), is a linear function of the forecast lead time, i.e., \( z_f(t) = bt + z_a \), where \( z_a \approx z_f(0) \) is the root-mean-square analysis error and the scalar \( b \) is the linear error growth rate. In contrast, the root-mean-square of the forecast error in the extra-tropics is an approximately exponential function of the forecast lead time, i.e., \( z_f(t) = z_a e^{rt} \), where the scalar \( r \) denotes the exponential error growth rate.

We obtain estimates of the parameters \( z_a \) and \( r \) by calculating their values for the curves that best fit the forecast errors for the first 72 hours in the least-squares sense. (These fitted curves are also shown in Figure 2.) Although the initial errors are very slightly larger in the SH extra-tropics than in the NH extra-tropics (0.42 ms\(^{-1}\) versus 0.39 ms\(^{-1}\)), the forecast errors grow a bit more slowly in the SH than in the NH extra-tropics; the error doubling time \( T = r^{-1} \ln 2 \) is 38.5 hours in the SH extratropics versus 34.7 hours in the NH extratropics.

We suggest that the tropical linear error growth rate \( b \) is directly associated with a
constant physical quantity that drives the evolution of \( z_f(t) \). One plausible candidate is that \( b \) reflects the rms of the generation rate of the convective available potential energy (CAPE). Thus, according to \( z_f(t) = bt + z_a \), the increase of the forecast error \( z_f(t) \) over the analysis error, \( z_f(0) = z_a \), is proportional to the CAPE generated by time \( t \). While a thorough verification of this conjecture is beyond the scope of the present study, it seems to be reasonably well supported by the observed evolution of the error fields. (i) A comparison of tropical error fields at consecutive forecast verification times reveals that, their spatially integrated magnitude is approximately the same at the two verification times, but that the spatial distribution of the errors themselves do change (the typical correlation between the error fields in the tropics at consecutive forecast verification times is 0.4). (ii) The differences between the error fields at consecutive times are dominantly small-scale features (not shown) in the regions of deep convection. Furthermore, the deep convective processes are parameterized by a modified Arakawa-Schubert scheme (Arakawa and Schubert 1974) in the NCEP GFS. This scheme is designed to balance the generation of CAPE in the large-scale flow with the consumption of CAPE by the convective processes. That is, while the convective processes have a local feedback to the large-scale flow, they do not change the total amount of CAPE that drives the future convective activity, if the generation of CAPE by diabatic forcing is unchanged. This explanation is also in line with the experience that a suitable diversity in an ensemble of forecasts in the tropics cannot be maintained without perturbing the diabatic forcing terms in the model equations (e.g., Buizza et al. 1999 and Puri et al. 2001).

Assume for the sake of argument that our conjecture is correct. Then one may ask whether the linear growth of the rms error in the tropics is an artifact associated with the convective parameterization scheme in the model, or whether one should expect similar behavior in the real atmosphere. Although the cloud model in the Arakawa-Schubert scheme may be unrealistic, the basic assumption regarding the balance of the generation and consumption of CAPE is strongly supported by observational evidence (Emanuel 1994). Thus, we would not be surprised if the rms distance between two nearby states of the real atmosphere grows
linearly in the tropics.

3.2 Dependence on the LEKF parameters

We have carried out experiments to test the sensitivity of the forecast results to the free parameters of the analysis scheme. (Results are shown only for the meridional component of the wind in the NH extra-tropics.) We find that, within a reasonable range of the parameters, the forecast errors depend only weakly on the parameters. More precisely, the small initial differences between the analyses for $5 \times 5 \times v$, $7 \times 7 \times v$, and $9 \times 9 \times v$ local cubes show negligible growth in the forecast phase (Figure 3). Likewise, for a $5 \times 5 \times v$ local region size, the advantage of the 80-member ensemble filter over the 40-member ensemble filter is negligible in the first 72 hours (Figure 4). Since the dominant errors grow exponentially in the extra-tropics, our result shows that differences in the analysis due to changes of the free parameters have only a very small projection on the dominant instabilities. This indicates that, when the parameters of the LEKF scheme are chosen from a reasonable range, the scheme can efficiently remove the growing error components. This is a nontrivial result, since the scheme corrects errors that were growing before the analysis time, while the forecast errors are governed by errors that are growing after the analysis time. An important practical consequence of the weak sensitivity to the tunable parameters is that it greatly increases the generality of our predictability assessment.

3.3 Dependence on the number of observations

In sharp contrast to the aforementioned weak sensitivity to the tunable parameters, the observational density has a significant influence on the accuracy of the forecasts. Increasing the number of observations substantially improves the accuracy of the forecasts in all geographical regions (results are shown only for the tropics and the NH extra-tropics). In the tropics, the improvement is essentially constant in time (Figure 5), due to the weak dependence of the linear error growth rate on the number of observations. This result suggests that increas-
ing the number of observations in the tropics leads to a reduction of the magnitude of the forecast errors, but it does not change the type of errors that dominate the error field. In the extra-tropics, the differences between the forecast errors for different observational densities grow exponentially, indicating that a larger number of observations has a greater impact on correcting the exponentially growing error components (Figure 6). On the other hand, the influence of the observational density on the exponential error growth rate is modest, although the error growth is slightly faster for the higher observational density (Table 1). For example, the largest local forecast errors at the 72-hour forecast lead time are about twice as large for the observational network that consists of only 500 soundings as compared to that for the one that observes all grid points.

3.4 Temporal variability of the forecast errors

Among the three geographical regions considered in this paper, the temporal variability of the forecast errors is the highest in the NH extra-tropics and the lowest in the tropics (Figure 7). We investigate the origin of the variability of the forecast errors with the help of Figure 8. This figure is obtained by normalizing the root-mean-square error at the different forecast lead times to their time averages and computing the standard deviation of the errors in the normalized time series. The larger variability of the NH extratropic forecast errors is clearly not due to larger variability of the analysis errors (shown as zero hour forecasts), but to an increase of variability between forecast lead times of 12 to 60 hours. This indicates that the larger variability of the forecast errors in the NH extratropics is due to a larger variability in the error growth rates between 12 and 60 hours lead times. We also note that the variability of the short term forecast errors (e.g., at 12 hours lead time) in Figure 7 is much smaller than in figure 5.6.1 of Kalnay (2003), in which are shown estimated forecast errors for a 3DVAR data assimilation system. This indicates that the LEKF efficiently reduces the “error of the day” component of the short-term forecast errors.

Figure 9, inspired by a graphical technique developed by Lorenz (2005), presents infor-
ation about the variability of the forecast errors in a format different from that of Figure 8. We consider forecasts started at analysis times \( t_a \) and verified at time \( t_v \); we normalize each realization of the forecast error by the sample mean of the errors of forecasts at forecast lead time \( t_v - t_a \); then we plot the normalized forecast errors using \( t_a \) and \( t_v \) as the two axes of the diagram. In this diagram values larger than 1 indicate unusually large errors at forecast lead time \( t_v - t_a \), while values smaller than 1 indicate unusually small errors at \( t_v - t_a \). In Figure 9 two interesting episode can be observed. The first episode is a pattern of extremely large errors in forecasts started between 1200 UTC on day 4 and 0000 UTC on day 7. The fact that the normalized errors are nearly constant in the horizontal \( (t_a) \) direction indicates that the unusually large forecast errors in this case are more associated with low predictability of the atmospheric states at \( t_v \) than with the accuracy of the analyses at \( t_a \). This conclusion is also supported by our finding (results not shown) that improving the the accuracy of the analysis, by adding more observations and/or increasing the ensemble size, leads to minuscule reductions in the normalized forecast errors at these verification times. An inspection of the atmospheric flow regimes reveals that the relatively low predictability of these states is associated with the rapid amplification of errors in the presence of an unusually strong jet stream in the North Atlantic storm track region (further details on this event are provided in sections 3.5 and 4.2). After day 7, the normalized forecast errors suddenly become nearly constant in the vertical \( (t_v) \) direction and rapidly decrease in the horizontal \( (t_a) \) direction. This indicates that between day-7 and day-8 observations of the unusually unstable states become available, leading to an efficient reduction of the analysis errors in the unusually unstable state space directions.

The second episode involves a pattern of unusually large analysis errors between about day 16 and day 24, that lead to a proportionally elevated level of forecast errors at the associated verification times. An inspection of the spatio-temporal evolution of the errors for this period (not shown) reveals that relatively large errors are due to exceptionally large analysis error in the region of Indonesia that later propagate to the NH extra-tropics. The visible propagation of the sample mean forecast errors from the tropics to the extra-tropics.
shown in Figure 1 is associated with this episode.

3.5 A case of explosive error growth

To gain a better understanding of the processes that lead to the explosive error growth in the aforementioned first episode, we select the forecast started at 1200 UTC on day 6 for further inspection. Maps of the forecast errors show that the explosive error growth at the 36-hour lead time occurs in a very localized region off the coast of Newfoundland (Figure 10). For the next 24 hours, the dominant error pattern is characterized by an eastward-propagating, rapidly-amplifying dipole structure. This structure and its fast propagation speed indicate that the dominant error pattern takes the shape of a packet of synoptic scale Rossby waves. This conclusion can be confirmed by calculating the packet envelope of the forecast errors for the 4- to 9-zonal-wavenumber range with a Hilbert transform-based method (Zimin et al. 2003 and 2005). Using the technique of Zimin et al. (2005), Figure 11 depicts an eastward-extending and amplifying wave packet envelope of errors. An inspection of the vertical cross-section of the errors (not shown) also confirms that the error growth starts in the jet layer with an overestimation of the wind speed in the core of the jet and a small distortion of the upper tropospheric wave near the core of the jet. Although downstream development (an initial divergence of the ageostrophic fluxes that triggers a baroclinic energy conversion; see Orlanski and Chang 1993 and Orlanski and Shelden 1995) leads to the development of a closed low associated with the upper tropospheric wave, the largest forecast errors occur further downstream, near the leading edge of the wave packet shown in Figure 11. Such propagation of the dominant errors was documented and analyzed in detail in Persson (2000), Szunyogh et al. (2000 and 2002), Zimin et al. (2003) and Hakim et al. (2005) and was foreseen long ago by the pioneers of numerical weather prediction (Rossby 1949; Charney 1949; Philips 1990).

In our example of rapid error growth, the atmospheric instability that drives the propagation of the errors, is a growing uncertainty in the characteristics (phase and amplitude) of
waves generated by an earlier downstream baroclinic development. The potential importance
of an instability process, in which an earlier baroclinic or barotropic instability leads to un-
certainties in the characteristics of developing waves, has been pointed out by Snyder (1999).
The fact that the dominant errors propagate along the upper tropospheric wave guides (Fig-
ure 1 and related discussion earlier) suggests that this may be the most important instability
when propagation of the forecast errors is concerned. The importance of this instability
process, in which temporal evolution and spatial propagation play equally important roles,
reinforces our view that the atmosphere should always be approached as a spatio-temporally
chaotic system.

4 The role of local low dimensionality

The E-dimension is a local, spatio-temporally evolving measure of complexity (Patil et al.
2001; Oczkowski et al. 2005). The calculation of this measure is based on the singular value
decomposition of an ensemble-based estimate of the analysis (or forecast) error covariance
matrix in a local region. Heuristically, the E-dimension measures the evenness of the distri-
bution of the estimated error variance between the different state space directions determined
by the eigenvectors of the covariance matrix. The lowest possible value of the E-dimension,
which is 1, occurs when the estimated variance is confined to a single state space direc-
tion. The highest possible value of the E-dimension, $N$, occurs when the variance is evenly
distributed between the $N$ state space directions.

As mentioned before, SEA05 found that the efficiency of the LEKF algorithm was in-
versonly related to the E-dimension. More precisely, a strong anti-correlation was found
between the E-dimension and that portion of the analysis error that lied in the subspace
spanned by the analysis ensemble perturbations. This relationship explained the finding
that the analysis errors were smallest in the regions of lowest E-dimensionality and largest
in the regions of highest E-dimensionality. In what follows, we demonstrate that the anti-
correlation between the E-dimension and the explained variance is even stronger for the
forecasts than for the analyses. In other words, we show that the lower the E-dimension, the more accurate the estimation of the uncertainty in the forecasts. This is an especially useful property of the ensemble, since, as shown in the next section, forecast errors grow fastest in the regions of lowest dimensionality.

4.1 Local low dimensionality and the evolution of forecast errors

While the choice of the coordinates of the state vector does not affect the state estimates, it has a profound effect on the singular value decomposition (SVD) of the error covariance matrices. Thus, the choice of coordinates has an important effect on such SVD-based diagnostics as the E-dimension. In this analysis, we use postprocessed output files whose horizontal resolution is 2.5 degrees in longitude and latitude. (The GFS resolution is approximately 1.875 degrees in longitude and latitude.) We follow the strategy of Oczkowski et al. (2005) and transform the ensemble perturbations so that the euclidean norm of the transformed perturbations has dimensions of energy. The local state vector is defined by all grid point variables in a local volume that contains a 5 × 5 horizontal grid (2.5-degree resolution) and the entire model atmosphere in the vertical direction. (The 5 × 5 postprocessed model grid contains about the same area as the 7 × 7 horizontal GFS grids used to obtain the forecasts.)

This definition of the local state vector differs from that used for the calculation of the E-dimension in SEA05. There, the local volume was defined by the local volume used in the LEKF algorithm, in which only a few model levels were included in the vertical direction and the number of model levels in the vertical layers was height dependent. The rationale for this change is that in SEA05, the goal was to evaluate the assumptions made in the implementation of the LEKF on the NCEP GFS; here, the goal is to study the role of local dimensionality in shaping the local predictability.

As expected based on the results of SEA05, the E-dimension is typically higher in the tropics (Figure 12) than in the extra-tropics. While the E-dimension decreases with increasing forecast time over the entire globe, the decrease of the dimension is much faster in the storm
track regions than elsewhere. Comparing Figures 1 and 12, we can draw the qualitative conclusion that the regions of the largest forecast error growth are the regions with the largest decrease of complexity.

One may wonder whether this effect is associated with an inherent property of the model dynamics or arises from an unexpected collapse of the ensemble due to some unforeseen problem with the ensemble generation technique. To answer this question, we apply the explained-variance diagnostic to the forecast error and the forecast ensemble. This diagnostic measures the square of the projection of the forecast error on the forecast ensemble normalized to the total forecast error variance. The close relationship between regions of low dimensionality and regions of high explained variance can be deduced both subjectively (comparing Figures 12 and 13) and objectively (Figure 14). The latter figure also shows that the close relationship strengthens with increasing forecast time.

4.2 Local low dimensionality and explosive local error growth

So far we have shown that there is a close relationship between the sample means of the forecast error, E-dimension and explained variance. A similarly close relationship exists between the spatio-temporally evolving fields of the same three quantities. Here we illustrate this close relationship on the example of the explosive forecast error growth describe in Section 3.5. In this case, the overlap between the regions of large errors and low dimensionality is almost perfect (Figure 15), especially at and after the 36-hour forecast lead time. Likewise, the explained variance rapidly grows in the regions of rapidly decreasing dimensionality, where the explained variance exceeds 90 percent at and beyond the 24-hour forecast lead time (Figure 16). Combining the results of the pictures, we conclude that in the regions of the fastest-growing errors, the ensemble performs extremely well in capturing the spatio-temporally evolving space of forecast uncertainties.
5 Conclusions

In this paper we assess atmospheric predictability with the help of a state-of-the-art numerical weather prediction model (at reduced resolution) and the Local Ensemble Kalman Filter data assimilation scheme. Our experimental design addresses the issue of determining the degree to which uncertainty in the knowledge of the initial state influences the predictability of a high-dimensional, spatio-temporally chaotic system. We assume that the numerical model provides a perfect representation of the true atmospheric dynamics. Our main findings are as follows:

- For this specific choice of the model and data assimilation system, the forecast errors grow exponentially in the extra-tropics and linearly in the tropics. As exponential growth has been found in many previous studies that considered different types of uncertainties in the knowledge of the true initial conditions, the dominance of exponentially-growing features seems to be an important property of predictability in the extra-tropics. Our research indicates that these dominant instabilities are closely related to the local generation and propagation of the eddy kinetic energy. Since these processes can be well-simulated by the models, there are good reasons to believe that exponentially-growing instabilities dominate real atmospheric dynamics in the extra-tropics. The linear growth of errors in the tropics is a more unique result of our experiments. While this result may be an artifact of the model dynamics, which rely heavily on parameterized physical processes in the tropics, we tend to believe that the real atmosphere behaves similarly.

- Local low dimensionality plays an important role in the evolution of uncertainties in the extra-tropics; the regions of local low-dimensionality and the regions of the fastest error growth coincide.

- A strong anti-correlation between the local dimensionality and the error variance explained by the LEKF ensemble is evident. This strong anti-correlation makes the LEKF
algorithm extremely efficient in estimating the analysis and forecast uncertainties in regions of local low dimensionality. The efficient estimation of the space of uncertainties enables the LEKF to produce very accurate estimates of both the initial states and the uncertainties in the forecast states. Although the unique features of the LEKF algorithm make the close relationship between local dimensionality, error growth, and explained error variance especially transparent, we believe that our results could be reproduced with any suitably formulated ensemble-based Kalman filter scheme (e.g., Anderson 2001; Bishop et al. 2001; Houtekamer and Mitchell 2001; Evensen 2003; Keppene and Rienecker 2002; Whitaker and Hamill 2002).

Do these results have any practical use when the forecast model is not perfect? First of all, it is safe to assume that the local dimensionality of the true atmosphere is higher than in our global forecast model. This would degrade the ability of the model-based ensemble to capture the space of forecast uncertainties. For instance, the strength of the anti-correlation between the E-dimension and the explained variance might be weaker, and the explained variance in the regions of low E-dimension might be lower. Our results suggest that if a weak anti-correlation is observed between the E-dimension and the explained variance, or if the explained forecast error variance is low in a region of local low dimensionality, then the likely culprit is model error. The properties of the LEKF method make it less likely that the small (40–100 member) ensemble has failed to capture instabilities that the model can resolve.

Local low dimensionality is a property that eventually breaks down with increasing forecast lead time. Eventually, predictability is completely lost, and the predictive value of the ensemble becomes the same as that of a set of randomly-drawn samples from the much larger set of climatologically realizable states of the model. The larger the magnitude of the initial ensemble perturbations, the earlier the breakdown of local low dimensionality occurs. For instance, Oczkowski et al. (2005) observed such breakdowns at forecast lead times of as little as 24 to 48 hours when investigating the evolution of a set of bred vectors. In our experimental design, the magnitude of the analysis uncertainty is small (presumably an order of magnitude smaller than in an operational weather analysis), so our results are not affected
by an overall breakdown of local low dimensionality in the first 120 hours of model integration. By extending the forecasts of the present study to longer forecast lead times we plan to investigate the process of the breakdown of low dimensionality in a future paper.

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## List of Figures

1. Sample mean error in forecasts of the meridional wind component at the 500 hPa pressure level at different forecast lead times. .............................................. 26

2. Dependence of the sample mean forecast error on the forecast lead time for the meridional wind component at the 500 hPa level in the three main geographical regions. Also shown are the curves, linear and exponential, fitted to the forecast errors of the first 72 hours. .............................................. 27

3. Dependence of the NH extratropics sample mean forecast error on the forecast lead time for three different choices of the local region size. Shown is the forecast error for the meridional wind component at the 500 hPa level on a semi-log scale. Also shown are the curves fitted to the forecast errors of the first 72 hours. .............................................. 27

4. Dependence of the NH extratropics sample mean forecast error on the forecast lead time for two different choices of the ensemble size. Shown is the forecast error for the meridional wind component at the 500 hPa level. The size of the local region is $5 \times 5 \times v$ for both curves. Also shown are the curves fitted to the forecast errors of the first 72 hours. .............................................. 28

5. Dependence of the NH-extratropics sample mean forecast error on the forecast lead time in the tropics for four different choices of the observational density. Shown is the forecast error for the meridional wind component at the 500 hPa level. Also shown are the curves fitted to the forecast errors of the first 72 hours. .............................................. 28

6. NH extratropics sample mean forecast error as function of the forecast lead time for four different choices of the observational density. Shown is the forecast error for the meridional wind component at the 500 hPa level. Also shown are the curves fitted to the forecast errors of the first 72 hours. .............................................. 29
7 Time series of the NH extratropics root-mean-square forecast error for different forecast lead times. Shown is the forecast error for the meridional wind component at the 500 hPa level. ................................. 30

8 Dependence of the variability of the root-mean-square forecast error on the forecast lead time in the NH extra-tropics. Shown is the forecast error for the meridional wind component at the 500 hPa level. ......................... 31

9 The color shades show the forecast error normalized by the sample mean forecast error at the associated forecast lead times (t_v - t_a). Results are shown only in a narrow diagonal band, since in our experimental design forecasts are available only up to 5 days. ................................. 32

10 Time evolution of the errors in the forecast started at 1200 UTC on day 7. Shown are the errors (color shades) and the “true” state of the geopotential height of the 500-hPa pressure level. ................................. 33

11 Time evolution of the wave packet envelope of errors in the forecast started at 1200 UTC on day 6. The wave packet envelope is calculated based on errors in the prediction of the meridional component of the wind vector in the zonal wavenumber range from 4 to 9. Notice the change in the color scheme between the 36-hour and 48-hour forecast lead times. ................................. 34

12 Sample mean E-dimension at different forecast lead times. ...................... 35

13 Sample mean explained variance at different forecast lead times. .............. 36

14 Sample mean explained variance vs. sample mean E-dimension at different forecast lead times. ................................................................. 37

15 Shown are the E-dimension (color shades) and the geopotential height forecast error at the 500 hPa level in the forecasts started at 1200 UTC on day 6. ................................. 38
Shown are the E-dimension (color shades) and the explained variance (contours) in the forecasts started at 1200 UTC day-6. The contour interval is 0.1 and values smaller than 0.7 are not shown.
Table 1: NH-extratropics root-mean-square analysis error, $z_a$, and error doubling time for the meridional wind component at the 500 hPa level at different observational densities. While these values are slightly different for the other model variables, they show the same tendencies.

<table>
<thead>
<tr>
<th>number of soundings</th>
<th>rms analysis error</th>
<th>error doubling time</th>
</tr>
</thead>
<tbody>
<tr>
<td>all locations</td>
<td>0.29 ms$^{-1}$</td>
<td>33.3 hours</td>
</tr>
<tr>
<td>2000 locations</td>
<td>0.39 ms$^{-1}$</td>
<td>34.7 hours</td>
</tr>
<tr>
<td>1000 locations</td>
<td>0.48 ms$^{-1}$</td>
<td>36.7 hours</td>
</tr>
<tr>
<td>500 locations</td>
<td>0.64 ms$^{-1}$</td>
<td>38.9 hours</td>
</tr>
</tbody>
</table>
Figure 1: Sample mean error in forecasts of the meridional wind component at the 500 hPa pressure level at different forecast lead times.
Figure 2: Dependence of the sample mean forecast error on the forecast lead time for the meridional wind component at the 500 hPa level in the three main geographical regions. Also shown are the curves, linear and exponential, fitted to the forecast errors of the first 72 hours.

Figure 3: Dependence of the NH extratropics sample mean forecast error on the forecast lead time for three different choices of the local region size. Shown is the forecast error for the meridional wind component at the 500 hPa level on a semi-log scale. Also shown are the curves fitted to the forecast errors of the first 72 hours.
Figure 4: Dependence of the NH extratropics sample mean forecast error on the forecast lead time for two different choices of the ensemble size. Shown is the forecast error for the meridional wind component at the 500 hPa level. The size of the local region is $5 \times 5 \times v$ for both curves. Also shown are the curves fitted to the forecast errors of the first 72 hours.

Figure 5: Dependence of the NH-extratropics sample mean forecast error on the forecast lead time in the tropics for four different choices of the observational density. Shown is the forecast error for the meridional wind component at the 500 hPa level. Also shown are the curves fitted to the forecast errors of the first 72 hours.
Figure 6: NH extratropics sample mean forecast error as function of the forecast lead time for four different choices of the observational density. Shown is the forecast error for the meridional wind component at the 500 hPa level. Also shown are the curves fitted to the forecast errors of the first 72 hours.
Figure 7: Time series of the NH extratropics root-mean-square forecast error for different forecast lead times. Shown is the forecast error for the meridional wind component at the 500 hPa level.
Figure 8: Dependence of the variability of the root-mean-square forecast error on the forecast lead time in the NH extra-tropics. Shown is the forecast error for the meridional wind component at the 500 hPa level.
Figure 9: The color shades show the forecast error normalized by the sample mean forecast error at the associated forecast lead times \((t_v - t_a)\). Results are shown only in a narrow diagonal band, since in our experimental design forecasts are available only up to 5 days.
Figure 10: Time evolution of the errors in the forecast started at 1200 UTC on day 7. Shown are the errors (color shades) and the “true” state of the geopotential height of the 500-hPa pressure level.
Figure 11: Time evolution of the wave packet envelope of errors in the forecast started at 1200 UTC on day 6. The wave packet envelope is calculated based on errors in the prediction of the meridional component of the wind vector in the zonal wavenumber range from 4 to 9. Notice the change in the color scheme between the 36-hour and 48-hour forecast lead times.
Figure 12: Sample mean E-dimension at different forecast lead times.
Figure 13: Sample mean explained variance at different forecast lead times.
Figure 14: Sample mean explained variance vs. sample mean E-dimension at different forecast lead times.
Figure 15: Shown are the E-dimension (color shades) and the geopotential height forecast error at the 500 hPa level in the forecasts started at 1200 UTC on day 6.
Figure 16: Shown are the E-dimension (color shades) and the explained variance (contours) in the forecasts started at 1200 UTC day-6. The contour interval is 0.1 and values smaller than 0.7 are not shown.