Geomagnetic Data Assimilation

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  - geodynamo
  - Differences between model output and observations
  - Motivation

• Mathematical Approach
  - Algorithm
  - Geomagnetic Assimilation System

• Results
  - Forecast results
  - Poloidal field: comparison with IGRF
  - Toroidal field and velocity field: changes due to data

• Discussion
Observed Geomagnetic field in the past 7000 years

Observed Br in 5000BC, Unit: nT
Geodynamo: Model

\[
\left( \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \mathbf{v} + 2 \Omega \times \mathbf{v} = -\nabla p + \frac{1}{\mu \rho_0} (\mathbf{B} \cdot \nabla) \mathbf{B} \\
+ \frac{\rho}{\rho_0} \mathbf{g} + \nu \nabla^2 \mathbf{v}
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}
\]
Geodynamo: graphic interpretation

(a single) Magnetic field line beneath the surface

(a single) Streamline in the outer core

Geodynamo from simulation
Geodynamo: Model

\[
\left( \frac{\partial}{\partial t} + v \cdot \nabla \right) v + 2\Omega \times v = -\nabla p + \frac{1}{\mu \rho_0} (B \cdot \nabla) B + \frac{\rho}{\rho_0} g + \nu \nabla^2 v
\]

\[
R_o \left( \frac{\partial}{\partial t} + v \cdot \nabla \right) v + \hat{z} \times v = -\nabla p + (B \cdot \nabla) B + R_{th} \delta \rho \mathbf{r} + E \nabla^2 v
\]

<table>
<thead>
<tr>
<th></th>
<th>( R_o )</th>
<th>( E )</th>
<th>( R_{th} )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Earth’s core</strong></td>
<td>( 10^{-9} )</td>
<td>( 10^{-15} )</td>
<td>?</td>
</tr>
<tr>
<td><strong>Simulation</strong></td>
<td>( 10^{-6} )</td>
<td>( 10^{-6} )</td>
<td>(~30 , R_c)</td>
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Comparison between the model output and observations

Observation

Pure dynamo simulation

Free model run $B_r$, 1990

Observed $B_r$, 1990
Comparison between the model output and observations

- Observations (for degree $L \leq 8$)
- Dynamo simulation from MoSST
- Non-dipolar part of the poloidal field at the CMB
Questions

Are we simulating geodynamo?

Can the observation be used to constrain the numerical dynamo models? i.e. bring numerical solutions closer to truth?

Can the improved model be able to reveal dynamical states in the core, and predict geomagnetic secular variation?

Very likely not

We intend to find out with data assimilation

We need to find out
Observation Record

• Observations are only at Earth’s surface (but we can estimate the value at the core mantle boundary).
• Good observation record is only about 400 years, while typical cycles are on the order of 100 years.
• Much older observations can be obtained from archeomagnetism and paleomagnetism.
Observations since 1500

Historical geomagnetic data

17th century data
geographical distribution

All figures are from Jonkers et al (2003)
Mathematical Approach: Sequential data assimilation

Geomagnetic data assimilation

\[
\frac{\partial x^f}{\partial t} = D(x^f, t)
\]
\[
x^f(t_0) = x_0
\]

Sequential Assimilation

\[
x^a = x^f(t_a^{(i)}) + K\{x^o(t_a^{(i)}) - Hx^f(t_a^{(i)})\}
\]
\[i = 0, 1, 2, \ldots\]

New Initial State

\[
x_0 = x^a, \quad t_0 = t_a^{(i)}
\]

Observations \(x^o\) are provided by field models

\[
\Delta t_a \equiv t_a^{(i+1)} - t_a^{(i)}, \text{ forecast time}
\]

State variables

\[ B = B_T + B_P \]
\[ = \nabla \times (T_B \hat{r}) + \nabla \times \nabla (P_B \hat{r}), \]
\[
\begin{bmatrix}
P_B \\
T_B
\end{bmatrix} = \sum_{0 \leq m \leq l} \begin{bmatrix}
b_l^m(r) \\
j_l^m(r)
\end{bmatrix} Y_l^m(\theta, \varphi) + C.C.,
\]
\[ B_r = \sum_{0 \leq m \leq l} \frac{l(l+1)}{r^2} b_l^m(r) Y_l^m(\theta, \varphi) + C.C. \]
Mathematical Approach: Algorithm

Covariance estimation

\[ x^a - x^f = K(x^o - Hx^f) \]

- **Analysis Increment calculated for each spectral coefficient separately**
- **Geomagnetic Observations downward to D''-layer in spectral space.**
- **Observation operator restricts the forecast to geomagnetic field at the core mantel boundary.**

\[ K = P^f H^t [H P^f H^t + R]^{-1} \]

- **Gain matrix uses standard OI minimum variance approach**
- **P^f is either modeled or Estimated from a long Model run**
Mathematical Approach: Algorithm

Covariance estimation

\[ P_f(L_1, m_1; L_2, m_2) = \frac{1}{N_{ens} - 1} \sum_{i=1}^{N_{ens}} \left[ \begin{array}{c}
(\tilde{P}_{L_1, m_1} - \tilde{\mu}_{P_{L_1, m_1}}) / \sigma_{P_{L_1, m_1}} \\
(\tilde{T}_{L_1, m_1} - \tilde{\mu}_{T_{L_1, m_1}}) / \sigma_{T_{L_1, m_1}} \\
(\tilde{P}_{L_2, m_1} - \tilde{\mu}_{P_{L_2, m_1}}) / \sigma_{P_{L_2, m_1}} \\
(\tilde{T}_{L_2, m_1} - \tilde{\mu}_{T_{L_2, m_1}}) / \sigma_{T_{L_2, m_1}} \\
(\tilde{\Theta}_{L_1, m_1} - \tilde{\mu}_{\Theta_{L_1, m_1}}) / \sigma_{\Theta_{L_1, m_1}} \\
(\tilde{\Theta}_{L_2, m_1} - \tilde{\mu}_{\Theta_{L_2, m_1}}) / \sigma_{\Theta_{L_2, m_1}} \\
(\tilde{\Theta}_{L_2, m_2} - \tilde{\mu}_{\Theta_{L_2, m_2}}) / \sigma_{\Theta_{L_2, m_2}} \\
(\tilde{\Theta}_{L_2, m_2} - \tilde{\mu}_{\Theta_{L_2, m_2}}) / \sigma_{\Theta_{L_2, m_2}} \\
\end{array} \right]^T \]
Mathematical Approach: Algorithm

Covariance estimation
Mathematical Approach: Algorithm

Optimal interpolation

```
x 10^-3

blm (L=m=1)

0

-1

-2

-3

0.85 0.9 0.95 1 1.05

Radius

Analysis

Forecast

ETH 2008

NASA
```
Results: Understanding the algorithms (Liu et al 2007)

Observing System Simulation Experiments (OSSEs)

Synthetic Observation: magnetic field from dynamo solutions at R = 15000

Model for assimilation: the same dynamo model at R = 14000
Results: Understanding the algorithms (Liu et al 2007)

Observing System Simulation Experiments (OSSEs)
Results: Understanding the algorithms (Liu et al 2007)

Observing System Simulation Experiments (OSSEs)
Results: assimilation with 7000 years of data

- Observation from GUFM1 (for degree \( L \leq 8 \))
- Dynamo simulation from MoSST
- Non-dipolar part of the poloidal field at the CMB
Results: assimilation with 7000 years of data
Impact of varying model parameters

- $R_{th} \approx 30 \, R_c$
- $R_{th} \approx 15 \, R_c$
- $R_{th} \approx 7.5 \, R_c$
**Discussion**

With current geomagnetic data assimilation system, MoSST_DAS, we are able to improve upon existing geomagnetic forecasts.

The unobserved variables inside the core are changed due to surface observations.

**Questions:**

- How can we get further improvements in forecasts?
- Are the changes to the unobserved variables an improvement?
- Can we use data assimilation to improve upon existing geodynamo models?
- Can we build an EnKF (or a hybrid) assimilation system?
assim-free: Horizontal of Velocity ($V_H$) ($r=1.0000$, year=2000AD)

assim07-100: Horizontal of Velocity ($V_H$) ($r=1.0000$, year=2000AD)