Comparison of 3D-Var and LETKF in an Atmospheric GCM: SPEEDY

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Outline

• SPEEDY
• Formulation
• Single Observation Experiments
• Observation Network Experiments
• Long-term Instabilities and Biases
• Summary and Future Work
SPEEDY (Molteni 2003)

• Model Description
  – Simplified Parameterizations, primitive-Equation DYNAMICS
  – Global atmospheric general circulation model of intermediate complexity
  – T30 spectral resolution – 96 x 48 grid points
  – 7 vertical levels (sigma coordinates)

• Experimental Setup
  – Output every 6 hours (Miyoshi 2005)
  – Experiments begin on January 1st, 1982 after 1 year of spin-up

<table>
<thead>
<tr>
<th>Observation Type</th>
<th>Observation Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>1 m/s</td>
</tr>
<tr>
<td>$v$</td>
<td>1 m/s</td>
</tr>
<tr>
<td>$T$</td>
<td>1 K</td>
</tr>
<tr>
<td>$q$</td>
<td>$10^{-4}$ kg/kg</td>
</tr>
<tr>
<td>$p_s$</td>
<td>100 Pa</td>
</tr>
</tbody>
</table>
Observation Networks

Dense Observation Network (1056 Stations)

Sparse Observation Network (264 Stations)

Realistic Observation Network (415 Stations)
3D-Var Formulation with Preconditioning

The analysis is obtained by finding the minimum ($\delta v^a$) of the cost function:

$$J(\delta v) = \frac{1}{2} \delta v^T \delta v + \frac{1}{2} \left( d^{ob} - H U_x \delta v \right)^T R^{-1} \left( d^{ob} - H U_x \delta v \right)$$

$$x^a = x^b + U_x \delta v^a$$

- $x^a$ – analysis ($\mathbb{R}^N$)
- $x^b$ – background ($\mathbb{R}^N$)
- $d^{ob}$ – innovation ($\mathbb{R}^L$)
- $H$ – linear observation operator ($\mathbb{R}^{L \times N}$)
- $R$ – observation error covariance matrix ($\mathbb{R}^{L \times L}$)
- $\delta v$ – preconditioned control vector ($\mathbb{R}^N$)

$$\delta x = U_x \delta v$$

$$B_x = U_x U_x^T \quad U_x = S_x F$$

- $S_x$ – background error standard deviation ($\mathbb{R}^{N \times N}$)
- $F$ – spatial correlations ($\mathbb{R}^{N \times N}$)

(recursive filter, Purser 2003a)
Local Ensemble Transform Kalman Filter (Hunt et al 2007)

- Analysis is computed locally for each grid point

\[ x_m^a = \bar{x}^a + \hat{X}^a \]

- ensemble mean \((\mathbb{R}^N)\)

\[ \bar{X} - \text{ensemble mean} \]

\[ \hat{X} = \{x_1 - \bar{x}, \ldots, x_M - \bar{x}\} - \text{ensemble spread} \]

\[ \bar{x}^a = \bar{x}^b + \hat{X}^b \bar{w} \]

where

\[ \bar{w} = (\tilde{P}^a)^{-1} (\hat{Y}^b)^T R^{-1} \text{d}^{ob} \quad (\mathbb{R}^M) \]

\[ \tilde{P}^a = (\tilde{P}^b)^{-1} + (\hat{Y}^b)^T R^{-1} \hat{Y}^b \quad (\mathbb{R}^{M\times M}) \]

\[ \hat{X}^a = \hat{X}^b W^a \]

where

\[ W^a = (\tilde{P}^a)^{1/2} \quad (\mathbb{R}^{M\times M}) \]

\[ \tilde{P}^b = (k - 1)^{-1} I \quad (\mathbb{R}^{M\times M}) \]
The horizontal velocity, $V$, can be broken down into its balanced and unbalanced components:

$$ V = rV^g + V^u $$

where

$V^g$ – geostrophic wind

$$ f k \times V^g = -RTV \ln(p_s) - \nabla \varphi(T) $$

$r$ – linear regression coefficient for $V$ and $V^g$

$$ r = \frac{E[(\varepsilon)(\varepsilon^g)^T]}{E[(\varepsilon^g)(\varepsilon^g)^T]} $$

$\varepsilon$ – difference in 18 and 24 hr forecasts of $V$ verifying at the same time (NMC method, Parrish and Derber 1992)

$r$ is computed so that $V^u$ and $V^g$ are statistically uncorrelated
To apply to 3D-Var, we transform the increment:

\[ \delta x = G U_z \delta z \]

where

- **G** – linearized geostrophic transformation
- **U_z** – background error for the \( \delta z \) rather than \( \delta x \)

The cost function becomes:

\[
J(\delta z) = \frac{1}{2} \delta z^T \delta z + \frac{1}{2} \left( d^{ob} - H G U_z \delta z \right)^T R^{-1} \left( d^{ob} - H G U_z \delta z \right)
\]
To apply to LETKF, we transform the ensemble to include the unbalanced wind:

$$x_m = g(z_m)$$

The analysis is performed on $z$ rather than $x$:

$$\bar{z}^a = \bar{z}^b + \hat{Z}^b W \quad \hat{Z}^a = \hat{Z}^b W^a$$

Variable localization

Removes correlation between $V^u$ and $(T, p_s)$

Employed through choice of observations used in local calculations
Single Observation - $T(\text{sig}=0.51)$

3D-Var,
Without Constraint

3D-Var,
With Constraint
Single Observation - $T(\text{sig}=0.51)$

**LETKF, Without Constraint**

**LETKF, With Constraint**
Comparison: 3D-Var

3D-Var
No Constraint

3D-Var
Constraint

Analysis RMSE, T(sig=0.51), 3D-Var, No Constraint

Analysis RMSE, T(sig=0.51), 3D-Var, Geostrophic

Dense
Sparse
Realistic
Free Run
Comparison: 3D-Var vs. LETKF

2-month 3D-Var and LETKF

Analysis RMSE for T(sig=0.51), Dense Obs Network

Analysis RMSE for T(sig=0.51), Sparse Obs Network

Analysis RMSE for T(sig=0.51), Realistic Obs Network

15-year Dense
No Constraint

3D-Var, No Geo
3D-Var, Geo
LETKF, No Geo
LETKF, Geo
Comparison: 3D-Var vs. LETKF

3D-Var, No Geo
3D-Var, Geo
LETKF, No Geo
LETKF, Geo

- Dense
- Sparse
- Realistic
Analysis Bias, T(sig=0.51), 02/01-03/01

Dense Network

No Constraint

Constraint

3D-Var

LETKF
Effect of Observation Networks

Randomizing Observation Locations for the 3D-Var, Dense, Geostrophic Constraint Case

Analysis, T(sig=0.51), Dense, Geo, 1982/08/09 00Z

Analysis RMSE for T(sig=0.51), 3D-Var, Dense, Geo

Regularly Spaced
Irregularly Spaced
Effect of Observation Networks

3D-Var, Sparse Network, Geostrophic Constraint

Analysis

Analysis, T(sig=0.51), Sparse, Geo, 1982/04/12 00z

Analysis Bias

Analysis Bias, T(sig=0.51), Sparse, Geo, 02/01–03/01
Bias: Spatial Pattern

Stationary waves

Wave length: 4 grid points

Observation locations occur between the crests and the troughs
There is a significant warm temperature bias, highest in the upper troposphere.
Why Assimilation Cannot Correct

Innovation Analysis

Increment
What Can We Do?

- Irregular observation locations
- Bias correction
- Change the length scale
- Reduce noise:
  - Digital filter
    - Spatial
    - Temporal not effective at removing biases stationary in time
  - Smoothing the background error extends the assimilation
  - Not assimilating $q$ observations above the 4$^{th}$ model level
    - The value of $q$ is equal or less than the observation error
What Can We Do?

Tune the Background Error
- Too small – Observations are not taken seriously enough, no convergence
- Too large – Observations are taken too seriously – sharp, large increments increase noise

![Graph of Analysis RMSE for T(sig=0.51), Dense, Geo, Tuning B](image1)

![Graph of Analysis RMSE for T(sig=0.51), Dense, Geostrophic](image2)
Summary

- 3D-Var and LETKF with and without the geostrophic constraint are implemented in the SPEEDY model.
- For each observational network and constraint option, LETKF outperforms 3D-Var.
- Biases and stability issues were encountered for the regularly spaced network cases when using the geostrophic constraint.
  - Currently under investigation.
  - Can be resolved by not using regularly spaced observations.
- Future work:
  - Hybrid 3D-Var/LETKF for SPEEDY
  - Evaluate for usefulness in the creation of a new reanalysis data set.
Thank You