The Rich-and-Poor model for Human and Nature Dynamics

Safa Motesharrei    Jorge Rivas    Eugenia Kalnay

University of Maryland

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  The Predator-Prey Model

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  Equilibrium Values and the Carrying Capacity

Scenarios
  Equitable Society: Rich = 0
  \( \kappa = 1 \) (Rich \( \geq 0 \))
  Rich \( \geq 0 \) and \( \kappa \gg 1 \)

Summary and Conclusions
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  Conclusions
Earth System Models have been developed since 1960s
- They started with just a model for the atmosphere
- New components like ocean, land, sea-ice, carbon cycle, vegetation, etc. were added over the past few decades
- However, the Human System is still a missing component
- Our goal: build a Human-Earth System model including all major feedbacks, such as climate change ⇔ population growth
The Predator-Prey Model

- A major inspirations for HANDY
- AKA Lotka-Volterra
- Derived independently by two mathematicians, Alfred Lotka and Vitto Volterra, in the early 20th century
- Describes the dynamics of competition between two species, say, wolves and rabbits
Predator-Prey System of Equations

The governing system of equations is

\[
\begin{align*}
\dot{x} &= (ay)x - bx \\
\dot{y} &= cy - (dx)y
\end{align*}
\]  \hspace{1cm} (1)

- \(x\): predator (wolf) population
- \(y\): prey (rabbit) population
- \(a\): determines the predator’s birth rate
- \(b\): the predator’s death rate
- \(c\): the prey’s birth rate
- \(d\): determines the predation rate
Equilibrium Points

The predator and prey populations show periodic, out of phase variations about the equilibrium values:

\[
\begin{align*}
    x_e &= \frac{c}{d} \\
    y_e &= \frac{b}{a}
\end{align*}
\] (2)
Typical Solution

Predator and Prey Populations

200 wolves
800 rabbits
150 wolves
600 rabbits
100 wolves
400 rabbits

0 100 200 300 400 500 600 700 800 900 1000
Time (year)

x Predator : Current
y Prey : Current
wolves
rabbits
An Overview of HANDY

- A minimalist coupled model
An Overview of HANDY

▶ A minimalist coupled model
▶ Explores the essential dynamics of interaction between population and natural resources
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- Portrays how different factors like inequality or high depletion rate can independently lead to a “Collapse”
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- Introduces “Carrying Capacity” as a measure for early detection of a collapse
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- Introduces “Carrying Capacity” as a measure for early detection of a collapse
- Offers a practical definition for carrying capacity
- Shows in order to reach an equilibrium, high unemployment is inevitable when productivity per worker is high
HANDY Equations

\[
\begin{align*}
\dot{x}_P &= \beta_P x_P - \alpha_P x_P \\
\dot{x}_R &= \beta_R x_R - \alpha_R x_R \\
\dot{y} &= \gamma y (\lambda - y) - \delta x_P y \\
\dot{w} &= \delta x_P y - C_R - C_P
\end{align*}
\]

- $x_R$: Rich — Non-working population (AKA Free-Riders)
- $x_P$: Poor — Labor force in charge of production
- $y$: Nature — An aggregate of physical and natural resources
- $w$: Wealth — Controlled by the Rich
Parameters and Functions in HANDY

- $\beta_R$ and $\beta_P$: Birth rates (constant) for the rich and the poor
- $\delta$: Productivity per capita
  AKA the “Production” or “Depletion” factor
- $\lambda$: Nature’s Capacity
- $\gamma$: Nature’s Regeneration rate
- $\kappa$: “Inequality” factor
- $C_R$ and $C_P$: Consumption rates for the Rich and the Poor
Consumption Rates in HANDY

- Consumption rates are given by
  \[
  \begin{align*}
  C_P &= \min \left(1, \frac{w}{w_{th}}\right) sx_P \\
  C_R &= \min \left(1, \frac{w}{w_{th}}\right) \kappa sx_R
  \end{align*}
  \]
  (4)

- \(s\): Consumption rate per capita, AKA the “Subsistence Salary”

- \(w_{th}\): a “Threshold” value for wealth below which famine starts, determined by the Rich

  \[w_{th} = \rho x_P + \kappa \rho x_R.\]
  (5)

- \(\rho\): minimum required consumption per capita
Death Rates in HANDY

- \( \alpha_R \) and \( \alpha_P \): Death rates for the Rich and the Poor

\[
\begin{align*}
\alpha_P &= \alpha_m + \max\left(0, 1 - \frac{C_P}{Sx_P}\right)(\alpha_M - \alpha_m) \\
\alpha_R &= \alpha_m + \max\left(0, 1 - \frac{C_R}{Sx_R}\right)(\alpha_M - \alpha_m)
\end{align*}
\] (6)

- \( \alpha_m \): Minimum (Normal) death rate
- \( \alpha_M \): Maximum death rate, prevails when \( w = 0 \)
- \( \alpha_R \) and \( \alpha_P \) can be equivalently expressed in terms of \( \frac{w}{W_{th}} \)
Equilibrium Values when $x_R = 0$

- Assume $\beta_R = \beta_P = \beta$
- $\beta$ satisfies $\alpha_m \leq \beta \leq \alpha_M$
- Define dimensionless $0 \leq \eta \triangleq \frac{\alpha_M - \beta}{\alpha_M - \alpha_m} \leq 1$
- Equilibrium values are then given by

\[
\begin{align*}
    x_{P,e} &= \frac{\gamma}{\delta} (\lambda - \eta \frac{s}{\delta}) \\
    y_e &= \eta \frac{s}{\delta} \\
    w_e &= \eta \rho x_{P,e}
\end{align*}
\]  

(7)
Carrying Capacity

Define $\chi$, the Carrying Capacity for population, to be equal to $x_{P,e}$

$$\chi \equiv \frac{\gamma}{\delta} \left( \lambda - \frac{s}{\delta \eta} \right)$$  \hspace{1cm} (8)
Define $\chi$, the Carrying Capacity for population, to be equal to $x_P,e$:

$$\chi \triangleq \frac{\gamma}{\delta} \left( \lambda - \frac{s}{\delta \eta} \right)$$  \hspace{1cm} (8)

$\chi$ is maximized when the Nature’s regeneration rate is maximal.
Carrying Capacity

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$\chi$ is maximized when the Nature’s regeneration rate is maximal

From $y_e = \frac{\lambda}{2}$, we can find the optimal $\delta$

$$\delta_* \triangleq \frac{2\eta s}{\lambda}$$  \hspace{1cm} (9)
Define $\chi$, the Carrying Capacity for population, to be equal to $\chi_{P,e}$

$$\chi \triangleq \frac{\gamma}{\delta} \left( \lambda - \frac{s}{\delta} \eta \right)$$  \hspace{1cm} (8)

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From $y_e = \frac{\lambda}{2}$, we can find the optimal $\delta$

$$\delta_* \triangleq \frac{2\eta s}{\lambda}$$  \hspace{1cm} (9)

This gives us the Optimal Carrying Capacity

$$\chi_M = \frac{\gamma \lambda}{\delta_* 2} = \frac{\gamma}{\eta s} \left( \frac{\lambda}{2} \right)^2$$  \hspace{1cm} (10)
Equilibrium when $x_R \geq 0$ and $\kappa = 1$

- Fix $\kappa \equiv 1$ to reach an equilibrium state for which $x_R \geq 0$
- Need a dimensionless free parameter $\varphi$ that fixes the ratio of the Rich to the Poor
  \[ \varphi = \frac{x_R}{x_P} \quad (11) \]
- Equilibrium values of the system can then be expressed as
  \[
  \begin{align*}
  x_{P,e} &= \frac{\gamma}{\delta} \left( \lambda - \eta \frac{s}{\delta} (1 + \varphi) \right) \\
  x_{R,e} &= \varphi x_{P,e} \\
  y_e &= \eta \frac{s}{\delta} (1 + \varphi) \\
  w_e &= \eta \rho (1 + \varphi) x_{P,e}
  \end{align*}
  \quad (12)
  \]
Maximum Equilibrium Population when $x_R \geq 0$ and $\kappa = 1$

- Total (equilibrium) population $x_e = x_{P,e} + x_{R,e}$ can be maximized by an appropriate choice of $\delta$

$$\delta_* = \frac{2\eta s}{\lambda} (1 + \varphi) \quad (13)$$

- Maximum total population is then given by

$$x_{e,M} = (1 + \varphi) \frac{\gamma \lambda}{\delta_*} \frac{\lambda}{2} = \frac{\gamma}{\eta s} \left(\frac{\lambda}{2}\right)^2 \quad (14)$$

- From (14), maximum total population is independent of $\varphi$ and conforms to the optimal carrying capacity given above by (10)
Soft Landing to the Optimal Equilibrium

Population, Nature, and Wealth

"kappa * x R equivalent rich population" : Current ppl
x P Poor Population : Current ppl
chi population carrying capacity : Current ppl
chi M optimal population carrying capacity : Current ppl
y Nature : Current eco$
w Accumulated Wealth : Current eco$
Oscillatory Approach to Equilibrium

Population, Nature, and Wealth

100,000 ppl
100 eco$
2,000 eco$
50,000 ppl
50 eco$
1,000 eco$
0 ppl
0 eco$
0 eco$

"kappa * x R equivalent rich population" : Current
x P Poor Population : Current
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Time (Year)
1750 1900 2050 2200 2350 2500 2650

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Equitable Society: \( \text{Rich} = 0 \)  
\( \kappa = 1 \) (\( \text{Rich} \geq 0 \))  
\( \text{Rich} \geq 0 \) and \( \kappa \gg 1 \)

Cycles of Prosperity and Collapse
Full Collapse

Equitable Society: Rich = 0
\( \kappa = 1 \) (Rich \( \geq 0 \))
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HANDY
Soft Landing to Equilibrium

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Oscillatory Approach to Equilibrium

Population, Nature, and Wealth

80,000 ppl
100 eco$
2,000 eco$
40,000 ppl
50 eco$
1,000 eco$
0 ppl
0 eco$
0 eco$

Time (Year)

1750 1900 2050 2200 2350 2500 2650

"kappa * x R equivalent rich population" : Current ppl
x P Poor Population : Current ppl
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Cycles of Prosperity and Collapse

Population, Nature, and Wealth

- Population: 80,000 ppl, 40,000 ppl, 0 ppl
- Nature: 100 eco$, 50 eco$, 0 eco$
- Accumulated Wealth: 2,000 eco$, 1,000 eco$, 0 eco$

Time (Year): 1750 to 2650

Equitable Society: Rich = 0
$\kappa = 1$ (Rich $\geq 0$)
Rich $\geq 0$ and $\kappa \gg 1$
**Full Collapse**

### Population, Nature, and Wealth

- **Population**
  - 80,000 ppl
  - 40,000 ppl
  - 0 ppl

- **Nature**
  - 2,000 eco$
  - 1,000 eco$
  - 0 eco$

- **Wealth**
  - 100 eco$
  - 50 eco$
  - 0 eco$

---

**kappa * x R equivalent rich population** : Current ppl
x P Poor Population : Current ppl
chi population carrying capacity : Current ppl
chi M optimal population carrying capacity : Current ppl
y Nature : Current eco$
w Accumulated Wealth : Current eco$
Preventing a Full Collapse by Increasing Unemployment

Population, Nature, and Wealth

"kappa * x R equivalent rich population" : Current
x P Poor Population : Current
chi population carrying capacity : Current
chi M optimal population carrying capacity : Current
y Nature : Current
w Accumulated Wealth : Current
Equitable Society: $\text{Rich} = 0$

$\kappa = 1$ ($\text{Rich} \geq 0$)

$\text{Rich} \geq 0$ and $\kappa \gg 1$

Population Collapse After an Established Equilibrium

**Diagram:**
- Population, Nature, and Wealth over time (Year)
- $600,000$ ppl, $100$ eco$, 400$ eco$
- $300,000$ ppl, $50$ eco$, 200$ eco$
- $0$ ppl, $0$ eco$, 0$ eco$

Variables:
- $\kappa$ * x R (equivalent rich population) : Current ppl
- $x$ P Poor Population : Current ppl
- $\chi$ Population carrying capacity : Current ppl
- $\chi M$ optimal population carrying capacity : Current ppl
- $y$ Nature : Current eco$
- w Accumulated Wealth : Current eco$

**Authors:** Safa Motesharrei, Jorge Rivas, Eugenia Kalnay

**Title:** HANDY
Cycles of Prosperity and Collapse

Equitable Society: $\text{Rich} = 0$

$\kappa = 1$ (Rich $\geq 0$)

Rich $\geq 0$ and $\kappa \gg 1$

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Population, Nature, and Wealth

Time (Year)

1750 1950 2150 2350 2550 2750 2950 3150 3350 3550 3750

"kappa * x R equivalent rich population" : Current
x P Poor Population : Current
chi population carrying capacity : Current
chi M optimal population carrying capacity : Current
y Nature : Current
w Accumulated Wealth : Current

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HANDY
**Equitable Society: Rich = 0**

\( \kappa = 1 \) (Rich \( \geq 0 \))

Rich \( \geq 0 \) and \( \kappa \gg 1 \)

### Full Collapse

**Population, Nature, and Wealth**

![Graph showing population, nature, and wealth over time](image)

- "kappa * x R equivalent rich population": Current population
- x P Poor Population: Current population
- chi population carrying capacity: Current population
- chi M optimal population carrying capacity: Current population
- y Nature: Current nature
- w Accumulated Wealth: Current accumulated wealth

**Years:** 1750, 1825, 1900, 1975, 2050, 2125, 2200

**Key:**
- ppl
- eco$

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Equilibrium can be reached (Soft Landing or Oscillatory Approach) if everyone works (Rich = 0).
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Equilibrium can be reached (Soft Landing or Oscillatory Approach) if everyone works (Rich = 0).

Collapse can happen if depletion per capita is too high even for the case Rich = 0.

Equilibrium can also be reached when Rich ≥ 0 but requires $\kappa = 1$. 
Conclusions

- Simple models provide a great intuition and can teach us invaluable points.
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- Simple models provide a great intuition and can teach us invaluable points
- It is crucial to have a measure that can give us an early warning for a collapse
- Carrying Capacity tells us when overshoot happens
- Carrying Capacity can be practically defined by noticing the decline in Wealth (Bubble Burst)
Simple models provide a great intuition and can teach us invaluable points.

It is crucial to have a measure that can give us an early warning for a collapse.

Carrying Capacity tells us when overshoot happens.

Carrying Capacity can be practically defined by noticing the decline in Wealth (Bubble Burst).

High unemployment and high production per capita are directly related.