Mixed Resolution
Ensemble Analysis

Assimilating with ensembles of differing accuracies.

by Sabrina Rainwater
Weather Prediction Perfection

• To perfectly predict the weather
  – exactly specify the current weather conditions,
  – perfectly model the atmospheric interactions.

• Unfortunately,
  – limited, imperfect observations,
  – simplified, numerical model.
Data Assimilation

• Data assimilation
  – estimate the current state of the weather.

• Keep it current,
  – we evolve it with a numerical model.
  – (the forecast phase)

• Keep it accurate,
  – we modify it every so often (usu. 6 hrs) to better match the most recent observations.
  – (the analysis phase)
Modifying the Forecast

• The Kalman Filter

\[ x_a = x_b + K \cdot (y_o - H \cdot x_b) \]
\[ K = P_b \cdot H^T \cdot (R_o + H \cdot P_b \cdot H^T)^{-1} \]

- \( x^{b/a} \) = background/analysis mean or best estimate
- \( H \) transforms model space to obs space, so:
- \( H \cdot x^b \) = background estimate in observation space
- \( P^b \) = BECM in model space
- \( y^o \) = observation vector
- \( R^o \) = OECM in obs space
- \( K \) = Kalman gain matrix
Kalman Filter Balance

Balancing new **observations** against the **forecast**, taking relative error into consideration.
What error does the forecast have?

• Ensemble answer:
  – evolve numerous weather states (instead of just one).

• Covariance estimate
  – estimated from statistics on the ensemble
    \[ P_b = \frac{1}{k-1} X_b \cdot X_b^T \]

• \( X_b \) = background ensemble in model space.
• \( k \) = the number of columns in \( X_b \)
  = the size of the ensemble.
Notation: $U_o$, $R_b$, $Y_b$.

$$U_o U_o^T = R_b = H \cdot P_b \cdot H^T$$

$$Y_b = H \cdot X_b$$

- $R_b$ is the background error covariance in observation space.
- $U_o$ is a root of $R_b$.
- $Y_b$ is the background ensemble in observation space.
- When possible, computations are performed in the (much smaller) observation space.
Mathematically Adapted ETKF

\[ U_o U_o^\top = R_b \]

1. \[ U_o = \frac{1}{\sqrt{k - 1}} Y_b \]

2. \[ C = U_o^\top R_o^{-1} \]

3. \[ M_a = \left( \frac{1}{\rho} I + C U_o \right)^{-1} \]

4. \[ X_a = X_b \left( M_a \right)^{\frac{1}{2}} \]

5. \[ x_a = x_b + \frac{1}{\sqrt{k - 1}} X_b M_a C \left( y_o - H x_b \right) \]

6. \[ X_{a,i} = x_a + X_{a,i} \]

• Step 1:
  – root covariance matrix.

• Steps 2 & 3:
  – efficient computation matrices.

• Steps 4, 5, & 6:
  – the analysis perturbations, mean, and ensemble.
Until Now …

• Ensemble data assimilation has only used
  – ensembles of the **same resolution**,  
  – ensembles with only **one high** resolution member and numerous low resolution members.

• We developed and tested a technique for performing ensemble data assimilation on **mixed resolution** ensembles

• Now ensembles are not limited to one high resolution member!
Modifying ETKF: Mixed $R_b$

- Main modification:
  - express $R_b$ as a weighted average
    $$R_b = \alpha \cdot R_b^l + (1-\alpha) \cdot R_b^h$$
- $R_b^{l/h} = \text{covariance estimated from low/high resolution subensemble.}$
  $$R_b^{l/h} = \frac{1}{\sqrt{k_{l/h} - 1}} Y_b^{l/h} \cdot \left( \frac{1}{\sqrt{k_{l/h} - 1}} Y_b^{l/h} \right)^T$$
- $Y_b^{l/h} = \text{the low/high resolution background ensemble in observation space.}$
- $k_{l/h} = \text{size of low/high ensemble}$
Rewrite \( R_b \)

- In ETKF we write
  \[
  R_b = \left( \frac{1}{\sqrt{k-1}} Y_b \right) \left( \frac{1}{\sqrt{k-1}} Y_b \right)^\top
  \]

- We can rewrite the mixed \( R_b \) similarly as a product of matrices.
  \[
  R_b = \left[ \sqrt{\frac{\alpha}{k_{\ell-1}}} Y_b^\ell : \sqrt{\frac{1-\alpha}{k_{h-1}}} Y_b^h \right] \left[ \sqrt{\frac{\alpha}{k_{\ell-1}}} Y_b^\ell : \sqrt{\frac{1-\alpha}{k_{h-1}}} Y_b^h \right]^\top
  \]

- This is mathematically equivalent to the weighted average on the previous slide.
Summary of New Notation for Mixed ETKF

- **R** = any error CM in obs sp
  - \( R_o \) = observation error CM
  - \( R_{b}^{h/l} \) = BECM in obs sp for h/l subensemble.
  - \( R_b \) = weighted average of \( R_{b}^{l} \) & \( R_{b}^{h} \).

- **U** = roots of ECMs
  - \( U_o \) = root of BECM in obs sp.
  - \( U_{h/l} \) = root of BEMC in high/low model sp.

- **spline** – interpolates low resolution vectors onto the high resolution space with a cubic spline

- **proj** – projects high resolution vectors onto the low resolution space.

- **Ensemble** = mean + pert
  - \( X_{a,i} = x_{a} + X_{a,i} \)
Mixed Step 1: Constructing $U_o$ and other root covariance matrices.

$$U_o = \left[ \sqrt{\frac{\alpha}{k_{\ell}-1}} Y_b^{\ell} : \sqrt{\frac{1-\alpha}{k_h-1}} Y_b^{h} \right]$$

(a) $U_h = \left[ \sqrt{\frac{\alpha}{k_{\ell}-1}} \cdot \text{spline}(X_b^{\ell}) : \sqrt{\frac{1-\alpha}{k_h-1}} \cdot X_b^{h} \right]$

(b) $U_ell = \left[ \sqrt{\frac{\alpha}{k_{\ell}-1}} \cdot X_b^{\ell} : \sqrt{\frac{1-\alpha}{k_h-1}} \cdot \text{proj}(X_b^{h}) \right]$

ETKF:

1. $U_o = \frac{1}{\sqrt{k-1}} Y_b$

Recall:
- $U =$ root of ECM
- $\text{spline}$ – interpolates low onto high
- $\text{proj}$ – projects high onto low
Mixed Steps 2 & 3: Creating C & $M^a$ for faster computation.

2. $C = U_o^\top R_o^{-1}$

3. $M_a = \left(\frac{1}{\rho}I + CU_o\right)^{-1}$

ETKF:

2. $C = U_o^\top R_o^{-1}$

3. $M_a = \left(\frac{1}{\rho}I + CU_o\right)^{-1}$

Note the equations are the same.
Mixed Step 4: Computing the analysis perturbations.

4. \[ \begin{bmatrix} X_A^\ell : X_A^h \end{bmatrix} = \begin{bmatrix} \text{spline} (X_b^\ell) : X_b^h \end{bmatrix} (M_a)^{\frac{1}{2}} \]

(a) \( X_a^h = X_A^h \)
(b) \( X_a^\ell = \text{proj} (X_A^\ell) \)

Recall:
• **spline** – interpolates low onto high
• **proj** – projects high onto low
Mixed Steps 5 & 6: Computing the analysis mean and analysis ensemble.

5. \( x_a = x_b + \frac{1}{\sqrt{k-1}} X_b M_a C (y_o - H x_b) \)

6. \( X_{a,i} = x_a + X_{a,i} \)

Done separately

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5. (a) \( x^h_a = x^h_b + U_h M_a C (y_o - H x^h_b) \)
   (b) \( x^\ell_a = x^\ell_b + U_\ell M_a C (y_o - H x^\ell_b) \)

6. (a) \( X^h_{a,i} = x^h_a + X^h_{a,i} \)
   (b) \( X^\ell_{a,i} = x^\ell_a + X^\ell_{a,i} \)
Models Used

• Lorenz Model II & Model III (2004)
• Model II = low resolution
• Model III = high resolution

• Model II is like Lorenz 96, but smoothed.
• Model III adds short wave coupling to Model II.
Parameters

- Circular grid of 960 units.
- One M3 grid point every 1 unit.
- One M2 grid point every 4 units.
- One observation every 24 units \( \approx 4\% \) grid points
- Obs Err of 2 (vs. 0.5)
- Forcing constants used: F=15, (F=14, F=12.5)
- 24 high res RK-integrations per \( dt=0.5 \) (vs. 12)
- Otherwise we followed the parameters described by Lorenz (2004)
Want to Show

• We want to show that mixed resolution ensemble analysis can improve both:
  – computation time (forecast time + analysis time)
  – analysis accuracy

• compared to
  – analysis with an ensemble at low resolution,
  – one high resolution member with a low resolution ensemble,
  – analysis with an ensemble at high resolution.
Ensembles Tested

- mLETKF tested with 30 members at low resolution and 1, 2, and 3 members at high resolutions (1&30, 2&30, 3&30)
- LETKF tested at low resolution with 15, 30, 45, 60, and 75 members
- LETKF tested at high resolution with 3, 4, 5, 6, 10, 15, and 30 members
Tuning

- tuned the covariance inflation, \( \rho \), in every case.
- Results insensitive to tuning \( \alpha \) (high/low weight).
- let \( \alpha = 0.5 \) in every mixed case.

Figure 2: Tuning \( \alpha \) for the 2 + 30 mixed case in the simple scenario. When \( \alpha = 1 \) the high resolution perturbations are ignored and only the low resolution perturbations are used to determine the analysis update. Similarly when \( \alpha = 0 \) the low resolution perturbations are ignored. The flatness of the graph indicates that \( \alpha \) does not need extensive tuning.
Comparing Time and Accuracy Chart

Small Ensembles

- Mixed: 1&30, 2&30, 3&30
- High: 3, 4, 5, 6
- Low: 15, 30, 45, 60, 75

Large Ensembles

RMS Error vs. Time per step

- Mixed
- High Res
- Low Res
## Comparing Time and Accuracy Table

<table>
<thead>
<tr>
<th>High &amp; Low</th>
<th>Time</th>
<th>RMS Error</th>
<th>Accuracy Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 &amp; 30</td>
<td>2.5</td>
<td>0.89</td>
<td>Low accuracy, fast</td>
</tr>
<tr>
<td>1 &amp; 60</td>
<td>4.5</td>
<td>0.88</td>
<td></td>
</tr>
<tr>
<td>6 &amp; 0</td>
<td>9.2</td>
<td>0.78</td>
<td>High accuracy, slow</td>
</tr>
<tr>
<td>4 &amp; 0</td>
<td>6.2</td>
<td>0.87</td>
<td></td>
</tr>
<tr>
<td>2 &amp; 30</td>
<td>4.1</td>
<td>0.81</td>
<td>Balanced</td>
</tr>
</tbody>
</table>
Conclusions

• We found that a mixed resolution analysis with multiple high resolution members can be more effective both in terms of speed and accuracy than the alternatives.
• If we only allowed one high resolution member and a low resolution ensemble, we could only increase accuracy by increasing the size of the ensemble, which has an upper limit of accuracy.
• Mixed resolution analysis increases this accuracy, while still running much faster than analysis on high resolution ensembles of similar accuracy.
• Thus, mixed resolution ensemble analysis is a better balance between computation time and accuracy than either high resolution analysis or analysis with one high resolution member and an ensemble of low resolution.
Acknowledgements

• Dr. Brian Hunt, my advisor in this research.
• Collin David, my husband and illustrator of Kalman Filter Balance.
## Time Results

<table>
<thead>
<tr>
<th>$k3, k2$</th>
<th>forecast time</th>
<th>analysis time</th>
</tr>
</thead>
<tbody>
<tr>
<td>0, 15</td>
<td>0.05</td>
<td>0.28</td>
</tr>
<tr>
<td>0, 30</td>
<td>0.09</td>
<td>0.72</td>
</tr>
<tr>
<td>0, 60</td>
<td>0.18</td>
<td>2.79</td>
</tr>
<tr>
<td>1, 30</td>
<td>1.58</td>
<td>0.95</td>
</tr>
<tr>
<td>2, 30</td>
<td>3.07</td>
<td>1.00</td>
</tr>
<tr>
<td>3, 30</td>
<td>4.56</td>
<td>1.05</td>
</tr>
<tr>
<td>3, 0</td>
<td>4.47</td>
<td>0.19</td>
</tr>
<tr>
<td>4, 0</td>
<td>5.96</td>
<td>0.20</td>
</tr>
<tr>
<td>5, 0</td>
<td>7.45</td>
<td>0.21</td>
</tr>
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<td>6, 0</td>
<td>8.94</td>
<td>0.22</td>
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<tr>
<td>10, 0</td>
<td>14.90</td>
<td>0.28</td>
</tr>
<tr>
<td>15, 0</td>
<td>22.35</td>
<td>0.37</td>
</tr>
<tr>
<td>30, 0</td>
<td>44.70</td>
<td>0.87</td>
</tr>
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Table 4: The time taken to compute the forecast and analysis in the simple scenario for various sizes of the high, low, and mixed resolution ensembles.
### Accuracy Results

**Time Averaged RMSE & Standard Deviation of the Fluctuations**

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<th>High Resolution Cases</th>
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<td>2 high</td>
</tr>
<tr>
<td>simple</td>
<td>0.89</td>
<td>0.81</td>
</tr>
<tr>
<td>shift obs</td>
<td>0.92</td>
<td>0.83</td>
</tr>
<tr>
<td>$F = 14$</td>
<td>0.94</td>
<td>0.86</td>
</tr>
<tr>
<td>$F = 12.5$</td>
<td>1.08</td>
<td>1.02</td>
</tr>
</tbody>
</table>

Table 1: The time averaged RMSE over 4500 time steps for various mixed and high resolution ensembles in various scenarios. These averages are accurate to at least the second decimal place. The standard deviation of the moving average in most cases was 0.02 or 0.03. The smallest (3 member) high resolution cases were not as precise, with a standard deviation of the moving average of 0.05 or 0.06.

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Table 2: The fluctuations in the RMSE with time. While the time averaged RMSEs are accurate to the hundredths place, the RMSE at any given time step is prone to fluctuate about that average. These fluctuations are listed in this chart.
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<tr>
<td>0, 45</td>
<td>0.14</td>
<td>1.60</td>
</tr>
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<td>0, 60</td>
<td>0.18</td>
<td>2.79</td>
</tr>
<tr>
<td>0, 75</td>
<td>0.23</td>
<td>4.67</td>
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Table 1: The time taken to compute the forecast and analysis in the simple scenario for various sizes of the high, low, and mixed resolution ensembles.
Table 2: RMSE mixed varying high and low ensemble size

<table>
<thead>
<tr>
<th>$k_2$</th>
<th>$k_3 = 1$</th>
<th>$k_3 = 2$</th>
<th>$k_3 = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1.00</td>
<td>0.87</td>
<td>0.81</td>
</tr>
<tr>
<td>10</td>
<td>0.93</td>
<td>0.83</td>
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<tr>
<td>20</td>
<td>0.91</td>
<td>0.82</td>
<td>0.79</td>
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Table 2: Varying the size of the low & high resolution ensembles. This table gives the RMS analysis error for the simple scenario with various mixed ensembles. A low resolution component with 10 members does nearly as well as 60.
# Tables 3 & 4:
Scenarios RMSE & Fluctuations

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<td>$F = 12.5$</td>
<td>1.08</td>
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Table 3: The time averaged RMSE over 4500 time steps for various mixed and high resolution ensembles in various scenarios. These averages are accurate to at least the second decimal place. The standard deviation of the moving average in most cases was 0.02 or 0.03. The smallest (3 member) high resolution cases were not as precise, with a standard deviation of the moving average of 0.05 or 0.06.

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Table 4: The fluctuations in the RMSE with time. While the time averaged RMSEs are accurate to the hundredths place, the RMSE at any given time step is prone to fluctuate about that average. These fluctuations are listed in this chart.
Table 5: Mixed is better than either component separately.

<table>
<thead>
<tr>
<th>Ensemble</th>
<th>RMSE</th>
<th>time</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 low</td>
<td>1.51</td>
<td>0.81</td>
</tr>
<tr>
<td>3 high</td>
<td>0.99</td>
<td>4.66</td>
</tr>
<tr>
<td>3 high + 30 low</td>
<td>0.79</td>
<td>5.62</td>
</tr>
<tr>
<td>6 high</td>
<td>0.78</td>
<td>9.16</td>
</tr>
</tbody>
</table>

Table 5: A comparison of the average RMSEs from data assimilation of two ensembles with different resolutions (3 high and 30 low), comparing LETKF on each separately with mLETKF on the combined ensemble, showing that mLETKF can use the less accurate information from the low resolution ensemble and still improve the high resolution ensemble. Only the results from the simple scenario are shown. The time required for one assimilation time step is listed in the far right column. The average RMSE of a 6 member high resolution ensemble is included for comparison with the 3 high and 30 low mixed resolution ensemble.
Figure 1: Sample Model 2 & Model 3 Output

Figure 1: Sample Model 2 and Model 3 states shown simultaneously. Model 3 has short wave coupling at high resolution, while Model 2, with a lower resolution, is smooth. One member of each ensemble is shown from a $1 + 30$ mixed run, simple scenario.
Figure 2: Dependence of RMS analysis error on $\alpha$ for the 2 & 30 mixed case in the simple scenario. When $\alpha = 1$ only the low resolution perturbations are used to determine the analysis update. The flatness of the graph indicates that $\alpha$ does not need extensive tuning.
Figure 3: Spin up High res vs Low res

Figure 3: Top: the RMSE of the analysis generated via LETKF with a low-resolution model, averaged over 4500 time steps after a 500 time step spin-up period, bottom: the RMSE of the analysis generated via LETKF with a high-resolution model, averaged over 4500 time steps after a 500 time step spin-up period.
Figure 4: The root mean square error at each time step for the initial 1000 time steps. This data is taken from the run of the 2 high resolution and 30 low resolution simple scenario.
Comparison to the usual $R^b$

Mixed $R^b$: \[
\widehat{R}^b = \left[ \sqrt{\frac{\alpha}{k_\ell - 1}} \bar{Y}^b_{\ell} : \sqrt{\frac{1 - \alpha}{k_h - 1}} \bar{Y}^b_h \right] \left[ \sqrt{\frac{\alpha}{k_\ell - 1}} \bar{Y}^b_{\ell} : \sqrt{\frac{1 - \alpha}{k_h - 1}} \bar{Y}^b_h \right]^T
\]

Usual $R^b$: \[
\widehat{R}^b = \left( \frac{1}{\sqrt{k-1}} \bar{Y}^b \right) \left( \frac{1}{\sqrt{k-1}} \bar{Y}^b \right)^T
\]

- If: \[k_\ell = \alpha k \text{ and } k_h = (1-\alpha)k\]

- then, \[\frac{\sqrt{1-\alpha}}{\sqrt{k_\ell - 1}} = \frac{\sqrt{\alpha}}{\sqrt{k_h - 1}} \approx \frac{1}{\sqrt{k-1}}\]

- So \[\widehat{R}^b \approx \left[ \frac{1}{\sqrt{k-1}} \bar{Y}^b_{\ell} : \frac{1}{\sqrt{k-1}} \bar{Y}^b_h \right] \left[ \frac{1}{\sqrt{k-1}} \bar{Y}^b_{\ell} : \frac{1}{\sqrt{k-1}} \bar{Y}^b_h \right]^T\]
Four Scenarios

- Simplest Case:
  \( F=15 \), observations lie on M2 grid points.

- Shifted Observations:
  \( F=15 \), observations lie halfway between M2 gps.

- Model Error (\( F=14 \)):
  \( F=14 \), observations lie on M2 grid points.

- Model Error (\( F=12.5 \)):
  \( F=12.5 \), observations lie on M2 grid points.
Complex Scenario Results

- The results in the more complex scenarios showed comparable improvement to that shown in the simple scenario.

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<tr>
<td></td>
<td>1 high +30 low</td>
<td>2 high +30 low</td>
</tr>
<tr>
<td>simple</td>
<td>0.89</td>
<td>0.81</td>
</tr>
<tr>
<td>shift obs</td>
<td>0.92</td>
<td>0.83</td>
</tr>
<tr>
<td>$F = 14$</td>
<td>0.94</td>
<td>0.86</td>
</tr>
<tr>
<td>$F = 12.5$</td>
<td>1.08</td>
<td>1.02</td>
</tr>
</tbody>
</table>

Table 1: The time averaged RMSE over 4500 time steps for various mixed and high resolution ensembles in various scenarios. These averages are accurate to at least the second decimal place. The standard deviation of the moving average in most cases was 0.02 or 0.03. The smallest (3 member) high resolution cases were not as precise, with a standard deviation of the moving average of 0.05 or 0.06.