Fallacies in Ensemble or Adjoint-Sensitivity Based Observation Impact Attributions

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Observation Impact Attributions

• Adjoint-based: Langland and Baker (2004)

OBS IMPACT: scalar example under BLUE

- Kalman filter

\[ x^a = x^f + \sigma_f^2 / (\sigma_f^2 + \sigma_o^2)(x^o - x^f) \]

\[ \sigma_a^2 = \sigma_o^2 \sigma_f^2 / (\sigma_f^2 + \sigma_o^2) \]

- Under Best Linear Unbiased Assumption (BLUE)

\[ \varepsilon^a = \varepsilon^f + \sigma_f^2 / (\sigma_f^2 + \sigma_o^2)(\varepsilon^o - \varepsilon^f) \]
OBS IMPACT: scalar example under BLUE

\[ \varepsilon^a = \varepsilon^f + \sigma_f^2 / (\sigma_f^2 + \sigma_o^2) (\varepsilon^o - \varepsilon^f) \]

If \( \varepsilon^o \varepsilon^f > 0 \) and \( |\varepsilon^o| > |\varepsilon^f| \),
we will have "bad" observation:
\( |\varepsilon^a| > |\varepsilon^f| \)

If \( \varepsilon^o \varepsilon^f < 0 \), we can still have "bad"
observation but much less likely

If \( \sigma^o \approx 0 << \sigma^f \), \( \sigma^a \approx \sigma^o = 0 \),
there will be almost no "bad" obs
OBS IMPACT: scalar example under BLUE

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OBS IMPACT: a multivariate example

• For a single obs $y$ w/o other obs at $t=0$

$$J^a = J^f + \frac{\text{cov}(y^f, J^f)}{\sigma^2_f + \sigma^2_o} (y^o - y^f)$$

• The impact of any obs $y_i$ among all obs:

$$J^a_t = J^a_{t,i} + \frac{\text{cov}(y_i^{a-i}, J^a_{t,i})}{\sigma^2_{a-i} + \sigma^2_o} (y^o_i - y^a_i)$$
OBS IMPACT: a multivariate example

\[ J_t^a = J_{t}^{a,-i} + \frac{\text{cov}(y_i^{a-i}, J_t^{a-i})}{\sigma_{a,-i}^2 + \sigma_o^2} (y_i^o - y_i^{a-i}) \]

Under BLUE assumptions, and if there is no error in the covariance estimate, the impact of any obs will again completely determined by the random error in the obs, and in the analysis w/o this analysis. There is again no need to say an obs is “good” or “bad” just because of random error in the analysis or observation.
OBS IMPACT: in an imperfect world

\[ J_t^a = J_t^{a,-i} + \frac{\text{COV}(y_i^{a-i}, J_t^{a-i})}{\sigma_{a,-i}^2 + \sigma_o^2} (y_i^o - y_i^a)^i \]

- Biases in either obs or first guess or \(-i\) analysis
- Error in the sampling covariance, versus true covariance due to:
  - Model error or bias, ensemble size or nonlearity or non-Gaussianity
  - Centering on \(y_f, y_a-\)i and \(J_f\), instead of \(y_t\) or \(J_t\)
OBS IMPACT: TC Example with position error
Sampling covariance vs. true covariance

- Even if the obs is perfect, the position error in prior (albeit random) can lead to the designation of this “perfect” obs as “bad”
Concluding Remarks on Observation Impact Attributions

• It is unclear what we gain from assigning impact of each individual observation, given it is primarily due to random error under BLUE
• A perfect obs can be “bad” because of error and bias in the prior or correlation; we shall not blame the perfect obs as being “bad” so to ignore or removing it but to use it for identifying forecast/analysis/model error, either random or systematic
• Nonlinearity and non-Gaussianity in correlation, model
• It matters which base trajectory (prior) that is used to derive the ensemble sensitivity or adjoint sensitivity that is used to assign observation impacts
• More physical meaning will be to see how much reduction in error covariance, even this can be skewed due to model error and bias
\[ P^b - P^a = P^b H^T (H P^b H^T + R)^{-1} HP^b \]

\[ P^b - P^a = X^b (H X^b)^T [H X^b (H X^b)^T + R]^{-1} H X^b (X^b)^T \]

\[ P^{\delta|b} - P^{\delta|a} \approx M X^b (H X^b)^T [H X^b (H X^b)^T + R]^{-1} H X^b (M X^b)^T \]

\[ P^{\delta|b} - P^{\delta|a} \approx J Y^T [Y Y^T + R]^{-1} Y J^T \]

\[ \delta J = J Y^T [Y Y^T + R]^{-1} [y - H(x^b)] \]