The Choice of Variable for Atmospheric Moisture Analysis

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Which variable to use for atmospheric moisture analysis?

| Met Office, Australian Bureau of Meteorology Research Centre | Relative Humidity |
| NCEP, ECMWF, Japan Meteorological Agency | Specific Humidity |
| Canadian Meteorological Centre, Naval Research Laboratory Atmospheric Variational Data Assimilation System | Logarithm of Specific Humidity |
Methodology

- Physical-Space/Finite-Volume Data Assimilation System (fvDAS)
- Data Assimilation Office NASA GFSC
- Finite-volume GCM (Lin and Rood 1996)
- Physical-Space Statistical Analysis System (PSAS)
- Observations:
  - Radiosonde soundings
  - Television Infrared Observation Satellite (TIROS) Operational Vertical Sounder (TOVS) retrievals
  - Total Precipitable water from Special Sensor Microwave Imager (SSM/I)
  - Progress to extract additional humidity information from radio occultation data transmitted by the Global Positioning System
Background

- The analysis is computed to minimize the cost function:
  \[ J(x) = (x^b - x)^T P^{-1} (x^b - x) + [y^o - h(x)]^T R^{-1} [y^o - h(x)] \]
- Assuming a linear observation operator, the minimizing solution is:
  \[ x^a = x^b + PH^T (HPH^T + R)^{-1} (y^o - H x^b) \]
- Structure of analysis increment \((x^a - x^b)\) strongly depends on specification of background error covariances
- In practice, background error covariances are not known and must be modeled
Background

- Ability to estimate error covariances is limited by lack of data
- Therefore, dealing with model errors presents a difficulty
- These issues are especially important for atmospheric moisture, which is strongly affected by mesoscale dynamics and nonlinear physical processes that are poorly resolved in GCMs
- Errors in model predictions of the humidity field can be quite large
Scientific Questions

• How robust is an analysis scheme in view of approximations in representation of errors?
• Which choice of atmospheric moisture control variable can provide meaningful estimates of the error covariances?
• Is it reasonable to characterize the error distribution by covariances?
Mixing Ratio and Specific Humidity

\[ w = \frac{m_v}{m_d} \]
\[ q = \frac{m_v}{(m_v + m_d)} = \frac{w}{1+w} \]

- Numerical values nearly identical and rarely exceed 20g/kg
- Extreme variability and changes in scale of errors and of the field itself

\[ w_j^a = w_j^b + \kappa (w_i^o - w_i^b) \]
\[ \kappa = \rho_{ij}\sigma_i^b\sigma_j^b/\{(\sigma_i^b)^2 + (\sigma_i^o)^2\} \]

- Meaningful extrapolation requires that error covariances accurately represent changes in errors between two points
Specific Humidity

Fig. 1. Specific humidity distribution at 0000 UTC 1 Jan 1998, produced by the ECMWF on a 55-layer $1^\circ \times 1.5^\circ$ lat-lon grid. The largest of the three panels shows the layer-mean specific humidity for the fourth model layer, where most of the water vapor tends to be concentrated (at approximately 850 hPa over the oceans). (top) Shows the vertical distribution along the equator in the lowest eight model layers, from the surface up to about 500 hPa. (right) Shows the vertical distribution along the Greenwich meridian.
Mixing Ratio Residuals:

\[ w^o - Hw^b = e^o - He^b \]
Log of Specific Humidity

\[ S = \log q \]

- Specific humidity of analyzed state always positive
- Singularity at \( q = 0 \)
- Analyzed humidity at a given location will approach zero whenever background or observation at that location is close to zero

\[
\begin{align*}
\log q_j^a &= \log q_j^b + \kappa (\log q_i^o - \log q_i^b) \\
q_j^a &= (q_j^b)^{1-\kappa} (q_i^o)^\kappa
\end{align*}
\]
Log of Specific Humidity Residuals

Fig. 3. As in Fig. 2 but for log residuals. Units are nondimensional.
Relative Humidity

\[ rh = \frac{w}{w_s(T,p)} \]

- Spatially and temporally more coherent
- Error statistics are easier to obtain
- Changes in temperature implied by the observations will cause the mixing ratio to adjust in such a way that the relative humidity background estimates remain unchanged
  - Used in Met Office, preferable for cloud parameterizations to maintain relative humidity by increasing specific humidity
  - If model has cool bias in stratosphere, then warming effect of temperature observations will cause spurious accumulation of moisture there
Relative Humidity Residuals

Fig. 4. As in Fig. 2 but for rh residuals. Units are in percents.
Pseudo-Relative Humidity

Mixing ratio is scaled by background saturation mixing ratio

\[ \tilde{w} = \frac{w}{w^{sb}} = \frac{w}{w^s(T^b, p)} \]

Background relative humidity and pseudo relative humidity are equal

\[ \tilde{w}^b = \frac{w^b}{w^{sb}} = (rh)^b. \]

Observed pseudo-relative humidity is not equal to the observed relative humidity

\[ \tilde{w}^o = \frac{w^o}{w^{sb}} \neq (rh)^o, \]

Amounts to being a flow-dependant transformation of the observed mixing ratio
Pseudo-Relative Humidity Residuals
A flow-dependant mixing ratio analysis

Variational analysis for mixing ratio is obtained by minimizing:

\[
J(w) = (w^b - w)^T P^{-1} (w^b - w) \\
+ [y^o - h(w)]^T R^{-1} [y^o - h(w)].
\]

Pseudo-relative humidity corresponds to a change of variable in model-state space and observation space

\[
\hat{w} = D^{-1} w, \quad D = \text{diag}(w^{ob}), \quad y^o = E^{-1} y^o, \quad E = \text{diag}[h(w^{ob})].
\]

Diagonal transformations \(D,E\) depend on the background temperature field and generate flow-dependant mixing ratio error variances

\[
\hat{P} = D^{-1} PD^{-1}, \quad \hat{R} = E^{-1} RE^{-1}.
\]

Pseudo-relative humidity analysis is equivalent to the analysis of mixing ratio with flow dependent variance specifications for both the background and observation errors

\[
J(\hat{w}) = (\hat{w}^b - \hat{w})^T \hat{P}^{-1} (\hat{w}^b - \hat{w}) \\
+ [\hat{y}^o - \hat{h}(\hat{w})]^T \hat{R}^{-1} [\hat{y}^o - \hat{h}(\hat{w})].
\]

where

\[
\hat{h}(\hat{w}) = E^{-1} h(D\hat{w}).
\]
Flow-dependant mixing ratio analysis

- Example of flow-dependant mixing ratio analysis that can occur in a pseudo-relative humidity analysis
- Vertical impact of a single moisture observation
- Dotted line marks value and location of observation residual
- Curves show vertical structure of mixing ratio increments $w_a - w_b$ obtained using a vertically homogeneous and isotropic pseudo-relative humidity error covariance model

Fig. 6. Mixing ratio analysis increments in a vertical column due to a single observation at 850, 500, 250, and 200 hPa, respectively, assuming vertically homogeneous and isotropic (in logge) pseudo-relative humidity error covariances. Saturation mixing ratios are computed based on a temperature profile with a lapse rate of 6 K km$^{-1}$ below 220 hPa, and constant temperature above. Temperature values are indicated along the vertical axis of the rightmost panel. The dotted lines mark the location and the value of the mixing ratio observation residual.
Multivariate moisture-temperature error correlations?

Fig. 7. Average time series correlations for mixing ratio and temperature differences (solid) and for relative humidity and temperature differences (dashed), computed from Dec 1999 quality-controlled radiosonde observed-minus-forecast residuals. The correlations were computed for each station at each mandatory level from the residual time series, and then averaged over all stations in the Northern Hemisphere, Tropics, and Southern Hemisphere.
Sensitivity to observations

Fig. 9. (top) The 500-hPa relative humidity observed-minus-background residuals (solid disks) and analysis increments (contours) obtained with a univariate relative humidity analysis, for the same region and time as in Fig. 8. (bottom) Implied mixing ratio analysis increments. Color shading is the same as in Fig. 8.

Fig. 8. (top) The 500-hPa mixing ratio observed-minus-background residuals (solid disks) and analysis increments (contours) obtained with a univariate pseudo-relative humidity analysis, for the eastern United States 0000 UTC 1 Jan 2002. (center) Temperature residuals and analysis increments. (bottom) Implied relative humidity analysis increments. The contour interval is indicated along the left axis; blue (red) shades represent negative (positive) values, and the shading of the disks is consistent with that of the contours.
Sensitivity to observations

- Difference between increments associated with the two analyses
- Relative humidity increments are equal to within 3% in most places
- Details of covariance models are less important with an abundance of observations
- Where data are sparse covariance specifications essentially determine how observational information is extrapolated to the model domain

Fig. 10. (top) Difference between the relative humidity increments shown in the top panel of Fig. 8 and the implied relative humidity analysis increments shown in the bottom panel of Fig. 9. (bottom) Difference between the implied mixing ratio analysis increments shown in the bottom panel of Fig. 8 and the mixing ratio analysis increments shown in the top panel of Fig. 9. Color shading is the same as in Fig. 8.
RMS mixing ratio background errors for fvDAS experiments

Red: Mixing ratio used for control variable
Blue: Impact of using pseudo-relative humidity as the control variable
Green: Added impact of TOVS retrievals

Fig. 11. Root-mean-square mixing ratio background errors for the three fvDAS experiments discussed in the text. Errors are calculated with respect to radiosonde mixing ratio observations in the Northern Hemisphere, Tropics, and Southern Hemisphere, at 1000, 850, 700, 500, and 300 hPa. The units along the horizontal axes are g kg⁻¹.
Future Work

- Optimal analysis scheme would require multivariate moisture-temperature error covariance specification
- May be feasible to improve the description of humidity errors by modeling three main dynamic effects on the background error covariances in the assimilation cycle:
  - advection of initial errors
  - error growth due to model defects
  - error reduction due to the incorporation of observation
Conclusions

• For mixing ratio and specific humidity error covariance modeling is complicated by the high variability of the errors and the field itself.
• The use of the log of specific humidity has the additional complication that a dry background is not well corrected by observations.
• Relative humidity is more coherent, but can lead to unrealistic and unstable stratospheric accumulation of moisture when model temperatures are biased.
• Pseudo relative humidity has similar statistical properties to relative humidity but preserves specific humidity in the absence of moisture observations.
• Easily implemented on existing moisture analysis systems by scaling observed-minus-background residuals prior to solving the analysis equation and then converting the analysis increments back to the original variable.
• fvDAS results indicate the change of variables leads to a better fit of background humidity estimates to radiosonde observations.