Satellite Retrieval
Data Assimilation

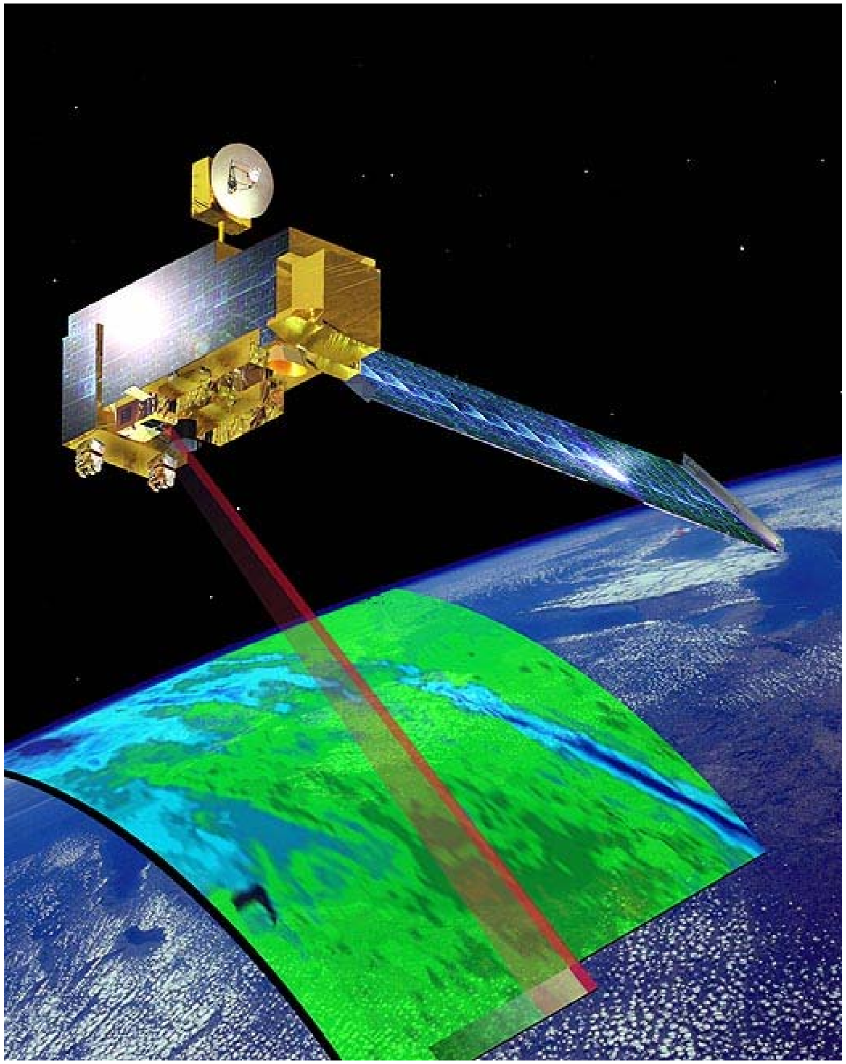
Rodgers, C. D.,

Inverse Methods for Atmospheric Sounding: Theory and Practice,

World Scientific Pub. Co., Hackensack, N.J., 2000

(Chapter 3 and Chapter 8)

Dave Kuhl



Artist depiction of NASA terra Satellite
with MOPITT instrument
http://www.space.gc.ca/asc/eng/apogee/2005/02_mopitt.asp

- **(Level 0)** Satellite instrument sensors unprocessed measurements.
- **(Level 1)** Unprocessed measurements are converted into radiances using instrument calibrated algorithms. In this step we have instrument errors, but no extra information is added in processing.
- **(Level 2)** Retrieval profiles of desired geophysical quantities (T,q,O_3,CO . . .) are calculated using retrieval algorithms based upon atmospheric physics. In this step *a priori* information must be introduced to constrain the ill posed inverse problem. This process introduces errors due to the physics as well as from the *a priori.*

The Problem

- Assimilating geophysical retrievals (which we will refer to a “**retrievals”**) containing *a priori* information introduces previously known information not related to the observation into our assimilation
  - In some retrievals the *a priori* information contained in a retrieval makes up the majority of the information -- In these cases you are then assimilating more to the *a priori* than to the observed quantity
  - The higher the percentage of information coming from the *a priori* in the retrieval the greater the problem
- Therefore we need to find a **new** method to remove the *a priori* information from the retrieval before we assimilate it

Organization of Talk

1. **Development of the Retrieval (the Rodgers Approach)**
2. **Error Analysis of the Retrieval**
3. **Discussion of a “new” retrieval method which accounts for this *a priori* information**

Development of Retrieval

observation
(i.e. radiances)

$$\mathbf{y} = \mathbf{f}(\mathbf{x}, \mathbf{b}) + \varepsilon$$

forward function
true physics

state vector
true state of the atmosphere
(i.e. T,q,O_3,CO)

forward function parameters
not included in the state but influence the measurement
(i.e. spectral line strengths)

error term
errors not related to the forward function parameters
(i.e. detector noise)

Development of Retrieval

$$\mathbf{y} = \mathbf{f}(\mathbf{x}, \mathbf{b}) + \varepsilon$$

observation
(i.e. radiances)

$$\mathbf{y} = \underbrace{\mathbf{F}(\mathbf{x}, \mathbf{b}')}_{} + \underbrace{\Delta_{\mathbf{F}}(\mathbf{x}, \mathbf{b}, \mathbf{b}')}_{} + \varepsilon$$

forward model error

forward model
an approximation of the true physics

forward function parameters used in the model
different due to the approximation of the physics

Development of Retrieval

$$\mathbf{y} = \mathbf{f}(\mathbf{x}, \mathbf{b}) + \varepsilon$$

$$\mathbf{y} = \mathbf{F}(\mathbf{x}, \mathbf{b}') + \Delta_{\mathbf{F}}(\mathbf{x}, \mathbf{b}, \mathbf{b}') + \varepsilon$$

Retrieval → $$\hat{\mathbf{x}} = \mathbf{R}(\mathbf{y}, \hat{\mathbf{b}}, \mathbf{c})$$

estimate of the state

(i.e. T,q,O_3,CO)

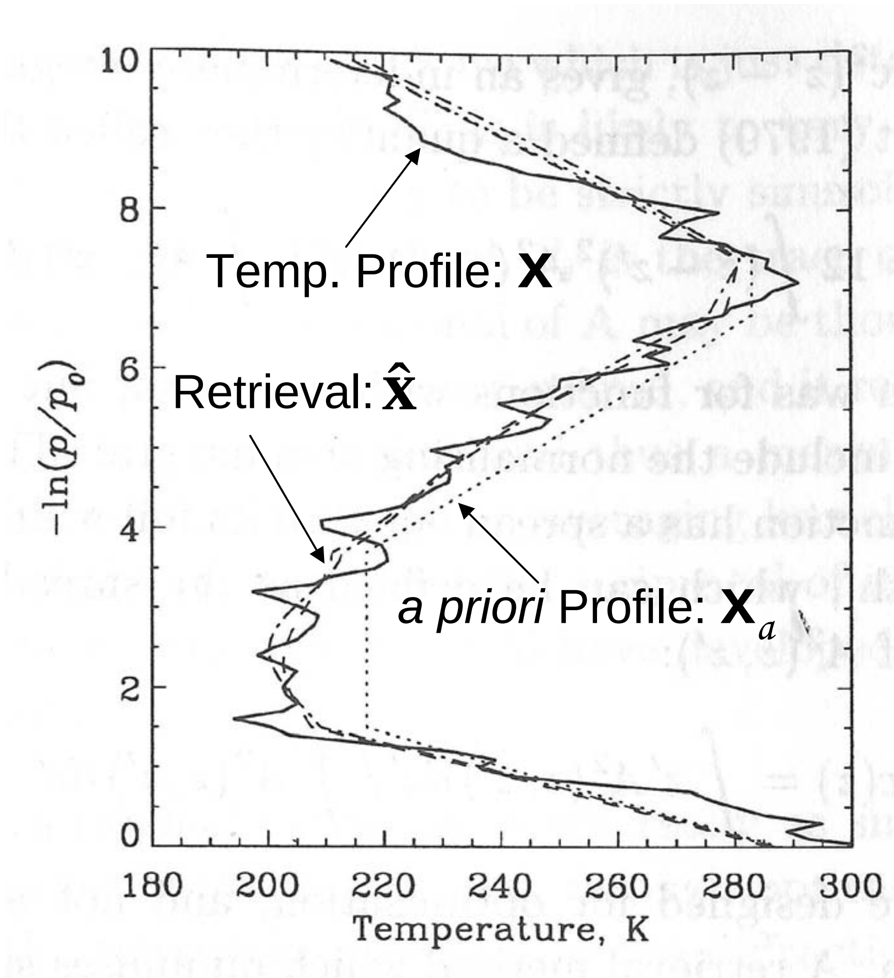
Inverse or Retrieval method

estimated forward function parameters

Parameters not in the forward function but affect retrieval for all reasonable inverse methods *a priori* should be the **only** parameter

Retrieval Theory

$$\hat{\mathbf{x}} = \mathbf{R}(\mathbf{y}, \hat{\mathbf{b}}, \mathbf{x}_a)$$



Rodgers 2000, Figure 3.3 p.56

Newly Added Slides

- I have added the next 16 slides to the presentation which describe the derivation of Rodgers Retrieval equation
- I did not show these in class due to time constraints

Newly Added Slides

Remote Measurement

observation
(i.e. radiances)

$$\mathbf{y} = \mathbf{f}(\mathbf{x}, \mathbf{b}) + \varepsilon$$

forward function
true physics

state vector
(i.e. T,q,O_3,CO)

forward function parameters
not included in the state
but influence the measurement
(i.e. spectral line strengths)

error term
errors not
related to the forward
function parameters
(i.e. detector noise)

Newly Added Slides

Remote Measurement

forward model error

$$\mathbf{y} = \mathbf{f}(\mathbf{x}, \mathbf{b}) + \varepsilon$$

$$\Delta_{\mathbf{F}} = \mathbf{f}(\mathbf{x}, \mathbf{b}) - \mathbf{F}(\mathbf{x}, \mathbf{b}')$$

forward function
true physics

forward function parameters used in the model
different due to the approximation of the physics

forward model
an approximation of the true physics

Newly Added Slides

Remote Measurement

$$\mathbf{y} = \mathbf{f}(\mathbf{x}, \mathbf{b}) + \varepsilon$$

$$\Delta_{\mathbf{F}} = \mathbf{f}(\mathbf{x}, \mathbf{b}) - \mathbf{F}(\mathbf{x}, \mathbf{b}')$$

$$\mathbf{y} = \mathbf{F}(\mathbf{x}, \mathbf{b}') + \Delta_{\mathbf{F}} + \varepsilon$$

observation
(i.e. radiances)

forward model
an approximation of the true physics

error term
errors not related to the forward function parameters (i.e. detector noise)

forward model error

Newly Added Slides

Remote Measurement

a priori
estimate of the state
unrelated to actual
measurement

$$\mathbf{y} = \mathbf{f}(\mathbf{x}, \mathbf{b}) + \varepsilon$$

$$\Delta_{\mathbf{F}} = \mathbf{f}(\mathbf{x}, \mathbf{b}) -$$

$$\mathbf{y} = \mathbf{F}(\mathbf{x}, \mathbf{b}') +$$

K_b: Sensitivity of Forward model to parameters

$$\mathbf{F}(\mathbf{x}, \mathbf{b}') = \mathbf{F}(\mathbf{x}_a, \hat{\mathbf{b}}) + \frac{\partial \mathbf{F}}{\partial \mathbf{x}}(\mathbf{x} - \mathbf{x}_a) + \frac{\partial \mathbf{F}}{\partial \mathbf{b}'}(\mathbf{b}' - \hat{\mathbf{b}})$$

estimated forward function parameters

K_x: Sensitivity of Forward model to state

Newly Added Slides

Remote Measurement

$$\mathbf{y} = \mathbf{f}(\mathbf{x}, \mathbf{b}) + \varepsilon$$

$$\Delta_{\mathbf{F}} = \mathbf{f}(\mathbf{x}, \mathbf{b}) - \mathbf{F}(\mathbf{x}, \mathbf{b}')$$

$$\mathbf{y} = \mathbf{F}(\mathbf{x}, \mathbf{b}') + \Delta_{\mathbf{F}} + \varepsilon$$

$$\mathbf{F}(\mathbf{x}, \mathbf{b}') = \mathbf{F}(\mathbf{x}_a, \hat{\mathbf{b}}) + \frac{\partial \mathbf{F}}{\partial \mathbf{x}}(\mathbf{x} - \mathbf{x}_a) + \frac{\partial \mathbf{F}}{\partial \mathbf{b}'}(\mathbf{b}' - \hat{\mathbf{b}})$$

$$\mathbf{F}(\mathbf{x}, \mathbf{b}') = \mathbf{F}(\mathbf{x}_a, \hat{\mathbf{b}}) + \mathbf{K}_{\mathbf{x}}(\mathbf{x} - \mathbf{x}_a) + \mathbf{K}_{\mathbf{b}'}(\mathbf{b}' - \hat{\mathbf{b}})$$

Newly Added Slides

Remote Measurement

$$\mathbf{y} = \mathbf{f}(\mathbf{x}, \mathbf{b}) + \varepsilon$$

$$\Delta_{\mathbf{F}} = \mathbf{f}(\mathbf{x}, \mathbf{b}) - \mathbf{F}(\mathbf{x}, \mathbf{b}')$$

$$\mathbf{y} = \mathbf{F}(\mathbf{x}, \mathbf{b}') + \Delta_{\mathbf{F}} + \varepsilon$$

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Newly Added Slides

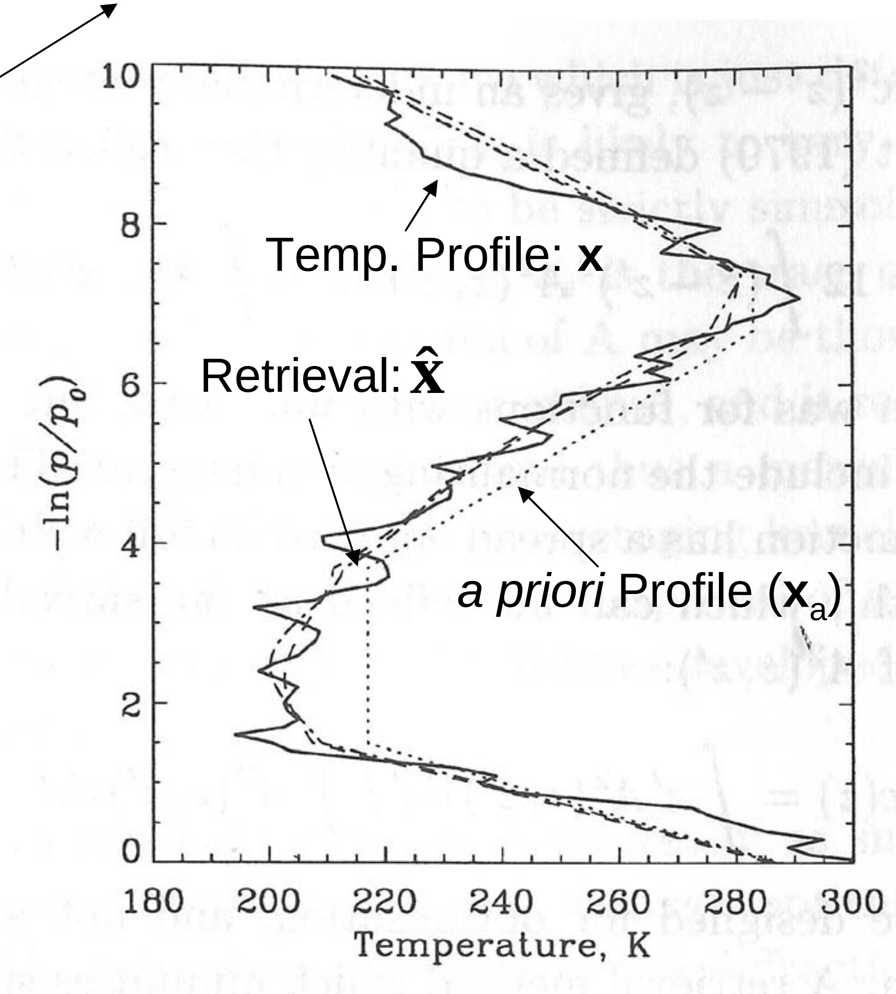
Retrieval Theory

Retrieval
estimate of the state
(i.e. T,q,O_3,CO)

$$\hat{\mathbf{x}} = \mathbf{R}(\mathbf{y}, \hat{\mathbf{b}}, \mathbf{c})$$

Inverse or Retrieval method

Parameters not in the forward function but affect retrieval for all reasonable inverse methods *a priori* should be the **only** parameter (**c**=$\mathbf{x}_a$)



Rodgers 2000, Figure 3.3 p.56

Newly Added Slides

Retrieval Theory

$$\hat{\mathbf{x}} = \mathbf{R}(\mathbf{y}, \hat{\mathbf{b}}, \mathbf{c})$$

$$\hat{\mathbf{x}} = \mathbf{R}(\mathbf{y}, \hat{\mathbf{b}}, \mathbf{x}_a)$$

Newly Added Slides

Retrieval Theory

forward model
of *a priori* and est.
forward funct. param.

$$\hat{\mathbf{x}} = \mathbf{R}(\mathbf{y}, \hat{\mathbf{b}}, \mathbf{c})$$

$$\hat{\mathbf{x}} = \mathbf{R}(\mathbf{y}, \hat{\mathbf{b}}, \mathbf{x}_a)$$

$$\hat{\mathbf{x}} = \underbrace{\mathbf{R}(\overbrace{\mathbf{F}(\mathbf{x}_a, \hat{\mathbf{b}})}, \hat{\mathbf{b}}, \mathbf{x}_a)} + \frac{\partial \mathbf{R}}{\partial \mathbf{y}}(\mathbf{y} - \mathbf{F}(\mathbf{x}_a, \hat{\mathbf{b}}))$$

For any well behaved
inverse method,
the method applied to the
a priori should yield
a priori as the result

G_y: Sensitivity of
Retrieval to measurement

Newly Added Slides

Retrieval Theory

$$\hat{\mathbf{x}} = \mathbf{R}(\mathbf{y}, \hat{\mathbf{b}}, \mathbf{c})$$

$$\hat{\mathbf{x}} = \mathbf{R}(\mathbf{y}, \hat{\mathbf{b}}, \mathbf{x}_a)$$

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$$\hat{\mathbf{x}} = \mathbf{x}_a + \mathbf{G}_{\mathbf{y}}(\mathbf{y} - \mathbf{F}(\mathbf{x}_a, \hat{\mathbf{b}}))$$

Newly Added Slides

Retrieval Theory

$$\hat{\mathbf{x}} = \mathbf{R}(\mathbf{y}, \hat{\mathbf{b}}, \mathbf{c})$$

$$\hat{\mathbf{x}} = \mathbf{R}(\mathbf{y}, \hat{\mathbf{b}}, \mathbf{x}_a)$$

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$$\hat{\mathbf{x}} = \mathbf{x}_a + \mathbf{G}_{\mathbf{y}}(\mathbf{y} - \mathbf{F}(\mathbf{x}_a, \hat{\mathbf{b}}))$$

$$\mathbf{y} = \mathbf{F}(\mathbf{x}_a, \hat{\mathbf{b}}) + \mathbf{K}_{\mathbf{x}}(\mathbf{x} - \mathbf{x}_a) + \mathbf{K}_{\mathbf{b}'}(\mathbf{b}' - \hat{\mathbf{b}}) + \Delta_{\mathbf{F}} + \varepsilon$$

Newly Added Slides

Retrieval Theory

$$\hat{\mathbf{x}} = \mathbf{R}(\mathbf{y}, \hat{\mathbf{b}}, \mathbf{c})$$

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$$\hat{\mathbf{x}} = \mathbf{x}_a + \mathbf{G}_{\mathbf{y}}(\mathbf{K}_{\mathbf{x}}(\mathbf{x} - \mathbf{x}_a) + \mathbf{K}_{\mathbf{b}'}(\mathbf{b}' - \hat{\mathbf{b}}) + \Delta_{\mathbf{F}} + \varepsilon)$$

Newly Added Slides

Retrieval Theory

$$\hat{\mathbf{x}} = \mathbf{x}_a + \mathbf{G}_{\mathbf{y}}(\mathbf{K}_{\mathbf{x}}(\mathbf{x} - \mathbf{x}_a) + \mathbf{K}_{\mathbf{b}'}(\mathbf{b}' - \hat{\mathbf{b}}) + \Delta_{\mathbf{F}} + \varepsilon)$$

Newly Added Slides

Retrieval Theory

$$\hat{\mathbf{x}} = \mathbf{x}_a + \mathbf{G_y}(\mathbf{K_x}(\mathbf{x} - \mathbf{x}_a) + \mathbf{K_{b'}}(\mathbf{b'} - \hat{\mathbf{b}}) + \Delta_\mathbf{F} + \varepsilon)$$

$$\hat{\mathbf{x}} = \mathbf{x}_a + \mathbf{G_y}\mathbf{K_x}(\mathbf{x} - \mathbf{x}_a) + \underbrace{\mathbf{G_y}\mathbf{K_{b'}}(\mathbf{b'} - \hat{\mathbf{b}}) + \mathbf{G_y}\Delta_\mathbf{F} + \mathbf{G_y}\varepsilon}$$

ε_y: all error terms

1. Forward Model Parameter
2. Forward Model
3. Detector Noise

Newly Added Slides

Retrieval Theory

$$\hat{\mathbf{x}} = \mathbf{x}_a + \mathbf{G_y}\left(\mathbf{K_x}(\mathbf{x} - \mathbf{x}_a) + \mathbf{K_{b'}}(\mathbf{b}' - \hat{\mathbf{b}}) + \Delta_{\mathbf{F}} + \varepsilon\right)$$

$$\hat{\mathbf{x}} = \mathbf{x}_a + \mathbf{G_y}\mathbf{K_x}(\mathbf{x} - \mathbf{x}_a) + \mathbf{G_y}\mathbf{K_{b'}}(\mathbf{b}' - \hat{\mathbf{b}}) + \mathbf{G_y}\Delta_{\mathbf{F}} + \mathbf{G_y}\varepsilon$$

$$\hat{\mathbf{x}} = \underbrace{\mathbf{G_y}\mathbf{K_x}}\mathbf{x} + (\mathbf{I} - \underbrace{\mathbf{G_y}\mathbf{K_x}})\mathbf{x}_a + \varepsilon_{\mathbf{y}}$$

A: Averaging Kernel
the sensitivity of true state to retrieval

$$\mathbf{A} = \mathbf{G_y}\mathbf{K_x} = \frac{\partial \mathbf{R}}{\partial \mathbf{y}}\frac{\partial \mathbf{F}}{\partial \mathbf{x}} = \partial \mathbf{x}' / \partial \mathbf{x}$$

Newly Added Slides

Retrieval Theory

$$\hat{\mathbf{x}} = \mathbf{x}_a + \mathbf{G}_{\mathbf{y}}(\mathbf{K}_{\mathbf{x}}(\mathbf{x} - \mathbf{x}_a) + \mathbf{K}_{\mathbf{b}'}(\mathbf{b}' - \hat{\mathbf{b}}) + \Delta_{\mathbf{F}} + \varepsilon)$$

$$\hat{\mathbf{x}} = \mathbf{x}_a + \mathbf{G}_{\mathbf{y}}\mathbf{K}_{\mathbf{x}}(\mathbf{x} - \mathbf{x}_a) + \mathbf{G}_{\mathbf{y}}\mathbf{K}_{\mathbf{b}'}(\mathbf{b}' - \hat{\mathbf{b}}) + \mathbf{G}_{\mathbf{y}}\Delta_{\mathbf{F}} + \mathbf{G}_{\mathbf{y}}\varepsilon$$

$$\hat{\mathbf{x}} = \mathbf{G}_{\mathbf{y}}\mathbf{K}_{\mathbf{x}}\mathbf{x} + (\mathbf{I} - \mathbf{G}_{\mathbf{y}}\mathbf{K}_{\mathbf{x}})\mathbf{x}_a + \varepsilon_{\mathbf{y}}$$

$$\hat{\mathbf{x}} = \mathbf{A}\mathbf{x} + (\mathbf{I} - \mathbf{A})\mathbf{x}_a + \varepsilon_{\mathbf{y}}$$

Development of Retrieval

Through a series of linearizations and an assumption we arrive at:

$$\begin{cases} \mathbf{y} = \mathbf{f}(\mathbf{x}, \mathbf{b}) + \varepsilon \\ \mathbf{y} = \mathbf{F}(\mathbf{x}, \mathbf{b}') + \Delta_{\mathbf{F}}(\mathbf{x}, \mathbf{b}, \mathbf{b}') + \varepsilon \\ \hat{\mathbf{x}} = \mathbf{R}(\mathbf{y}, \hat{\mathbf{b}}, \mathbf{x}_a) \end{cases}$$

ε_y: error terms

1. Forward Model Parameter
2. Forward Model
3. Detector Noise

$$\hat{\mathbf{x}} = \mathbf{x}_a + \frac{\partial \mathbf{R}}{\partial \mathbf{y}} \frac{\partial \mathbf{F}}{\partial \mathbf{x}} (\mathbf{x} - \mathbf{x}_a) + \overbrace{\frac{\partial \mathbf{R}}{\partial \mathbf{y}} \left(\frac{\partial \mathbf{F}}{\partial \mathbf{b}'} (\mathbf{b}' - \hat{\mathbf{b}}) + \Delta_{\mathbf{F}} + \varepsilon \right)}$$

Sensitivity of the Retrieval to measurement

Sensitivity of Forward model to parameters

Development of Retrieval

$$\hat{\mathbf{x}} = \mathbf{x}_a + \underbrace{\frac{\partial \mathbf{R}}{\partial \mathbf{y}} \frac{\partial \mathbf{F}}{\partial \mathbf{x}}} (\mathbf{x} - \mathbf{x}_a) + \varepsilon_{\mathbf{y}}$$

Sensitivity of the Retrieval to measurement (→ $\frac{\partial \mathbf{R}}{\partial \mathbf{y}}$)

Sensitivity of the Forward Model to the state (→ $\frac{\partial \mathbf{F}}{\partial \mathbf{x}}$)

Averaging Kernel is the sensitivity of the retrieval to the true state

$$\mathbf{A} = \frac{\partial \mathbf{R}}{\partial \mathbf{y}} \frac{\partial \mathbf{F}}{\partial \mathbf{x}} = \frac{\partial \hat{\mathbf{x}}}{\partial \mathbf{x}}$$

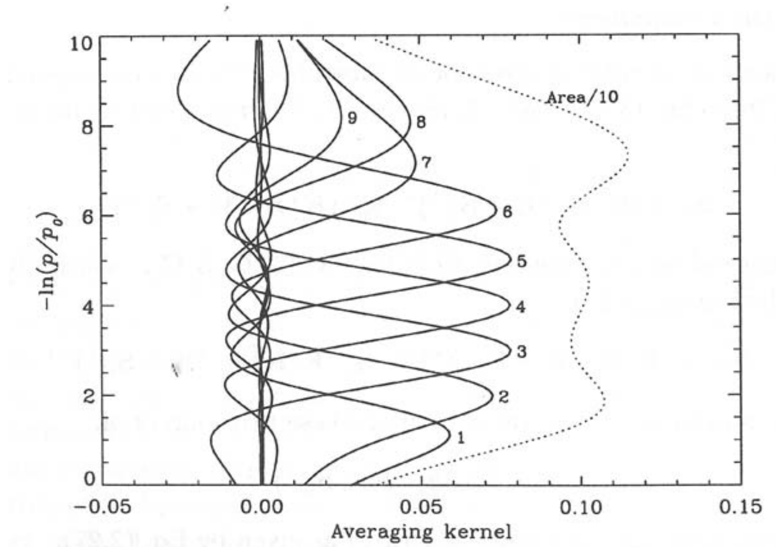
$$\mathbf{A} = \begin{matrix} a_{1,1} & \cdots & a_{n,1} \\ \vdots & \ddots & \vdots \\ a_{1,n} & \cdots & a_{n,n} \end{matrix}$$

$$\begin{bmatrix} \hat{x}_{(1)} \\ \vdots \\ \hat{x}_{(n)} \end{bmatrix} = \begin{bmatrix} x_{a(1)} \\ \vdots \\ x_{a(n)} \end{bmatrix} + \begin{bmatrix} a_{1,1} & \cdots & a_{n,1} \\ \vdots & \ddots & \vdots \\ a_{1,n} & \cdots & a_{n,n} \end{bmatrix} \left(\begin{bmatrix} x_{(1)} \\ \vdots \\ x_{(n)} \end{bmatrix} - \begin{bmatrix} x_{a(1)} \\ \vdots \\ x_{a(n)} \end{bmatrix} \right) + \begin{bmatrix} \varepsilon_{\mathbf{y}(1)} \\ \vdots \\ \varepsilon_{\mathbf{y}(n)} \end{bmatrix}$$

Closer look at Averaging Kernel

- The Rows of the Kernel
  - the retrieval at any level is an average of the whole profile weighted by this row.
  - An ideal inverse function would have **A**=**I**
- The area of the Kernel
  - Sensitivity of retrieval to true profile
  - Close to unity indicates high sensitivity
- The Columns of the Kernel
  - the response of the observing system to a δ-function disturbance at that retrieval level.

$$\begin{bmatrix} a_{1,1} & \cdots & a_{n,1} \\ \vdots & \ddots & \vdots \\ a_{1,n} & \cdots & a_{n,n} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$



Rodgers 2000, Figure 3.5 p.57

Development of Retrieval

$$\hat{\mathbf{x}} = \mathbf{x}_a + \mathbf{A}(\mathbf{x} - \mathbf{x}_a) + \varepsilon_{\mathbf{y}}$$

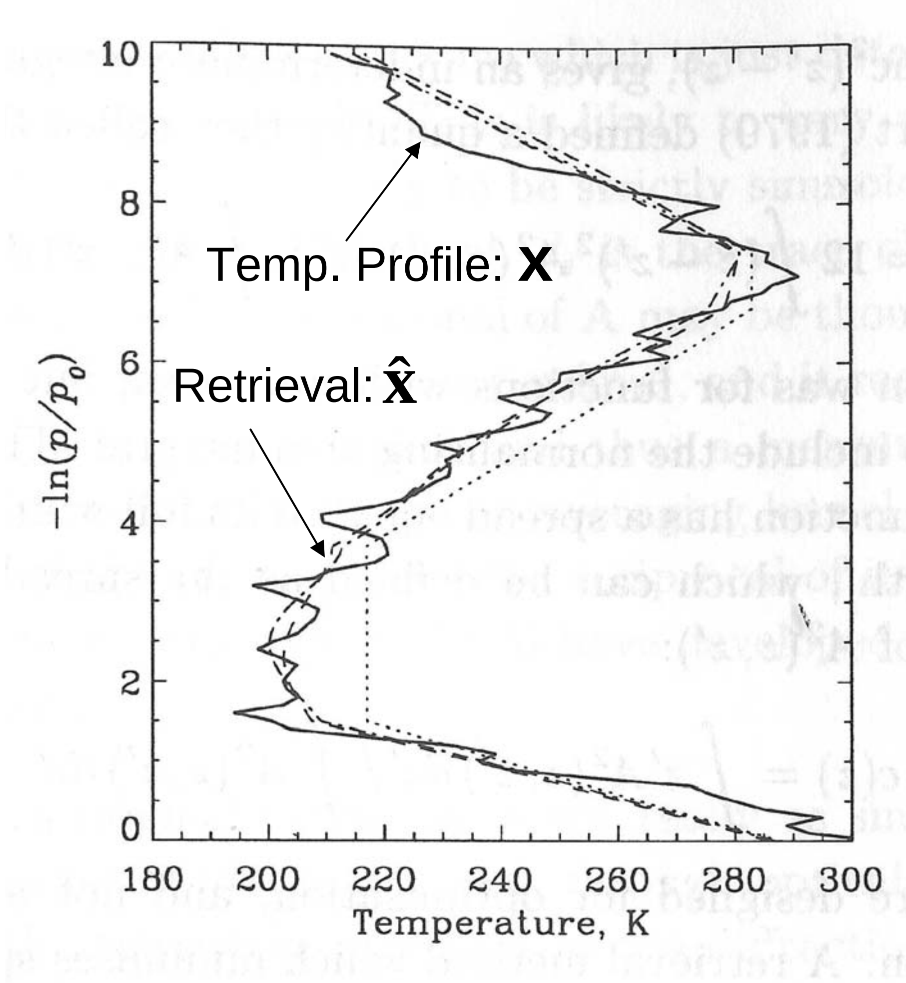
More Useful Form for error analysis

$$\hat{\mathbf{x}} = \mathbf{A}\mathbf{x} + (\mathbf{I} - \mathbf{A})\mathbf{x}_a + \varepsilon_{\mathbf{y}}$$

Error Analysis

$$\hat{\mathbf{x}} - \mathbf{x} = \mathbf{A}\mathbf{x} - \mathbf{x} + (\mathbf{I} - \mathbf{A})\mathbf{x}_a + \varepsilon_{\mathbf{y}}$$

- We subtract the true profile since we want to know how close to the retrieval is to the truth.
- This is useful to us since traditionally the assimilation compared the model generated profile to the retrieved profile directly.



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Error Analysis

$$\hat{\mathbf{x}} - \mathbf{x} = \mathbf{A}\mathbf{x} - \mathbf{x} + (\mathbf{I} - \mathbf{A})\mathbf{x}_a + \varepsilon_{\mathbf{y}}$$

$$\hat{\mathbf{x}} - \mathbf{x} = (\mathbf{A} - \mathbf{I})(\mathbf{x} - \mathbf{x}_a) + \frac{\partial \mathbf{R}}{\partial \mathbf{y}} \frac{\partial \mathbf{F}}{\partial \mathbf{b}'}(\mathbf{b}' - \hat{\mathbf{b}}) + \frac{\partial \mathbf{R}}{\partial \mathbf{y}} \Delta_{\mathbf{F}}(\mathbf{x}, \mathbf{b}, \mathbf{b}') + \frac{\partial \mathbf{R}}{\partial \mathbf{y}} \varepsilon$$

$$\varepsilon_{\hat{\mathbf{x}}} = \varepsilon_s + \varepsilon_f + \varepsilon_F + \varepsilon_m$$

Smoothing	Forward Model Parameters	Forward Model	Retrieval Noise

What we need to investigate for each term:

1. The mean error $\bar{\varepsilon}$
2. The error covariance S

Mean Smoothing Error

$$\varepsilon_s = (\mathbf{A} - \mathbf{I})(\mathbf{x} - \mathbf{x}_a)$$

•Because the true state is not normally known we cannot estimate the actual smoothing error

•What we require is a description of the statistics of the error, which must be calculated from the mean and covariance over some appropriate ensemble of states

$$\overline{\varepsilon_s} = (\mathbf{A} - \mathbf{I})(\overline{\mathbf{x}} - \mathbf{x}_a) = 0 \quad \textbf{if} \quad \overline{\mathbf{x}} = \mathbf{x}_a$$

•This would occur if the *a priori* is equal to the mean value of an ensemble of states for all profiles

We will assume this term to be small

Note: this assumption can fail depending upon the *a priori*

Smoothing Error Covariance

Expected Value Operator

$$\mathbf{S}_s = E\{\varepsilon_s \varepsilon_s{}^{\mathrm{T}}\}$$

$$\mathbf{S}_s = (\mathbf{A} - \mathbf{I})\mathbf{S}_a(\mathbf{A} - \mathbf{I})^{\mathrm{T}}$$

$$\mathbf{S}_a = E\{(\mathbf{x} - \mathbf{x}_a)\cdot(\mathbf{x} - \mathbf{x}_a)^{\mathrm{T}}\}$$

- Where $\mathbf{S}_a$ is the *a priori* error covariance matrix
  - we are sometimes given
  - or we can calculate it with other information

We will NOT assume this term to be small

Mean Model Parameter Error

$$\varepsilon_f = \frac{\partial \mathbf{R}}{\partial \mathbf{y}} \frac{\partial \mathbf{F}}{\partial \mathbf{b}'} (\mathbf{b}' - \hat{\mathbf{b}})$$

- The error in the retrieval due to errors in the forward model parameters
- Straightforward to evaluate:
  - If the forward model parameters have been estimated properly
  - and the model is linear as far as they are concerned
- The sensitivity of the retrieval to the measurement
  - derive algebraically
  - or perturbation from the inverse method
- The sensitivity of the forward model to the forward model parameters.
  - derive algebraically
  - or perturbation from the forward model
- If it is a **good retrieval algorithm** then the errors will be unbiased and the assumed **model parameter error will be small**

We will assume this term to be small

Note however if the satellite instrument changes or if the instrument is not fully characterized this assumption is wrong.

Model Parameter Error Covariance

$$\mathbf{S}_f = E\{\varepsilon_f \varepsilon_f{}^{\mathrm{T}}\}$$

$$\mathbf{S}_f = \frac{\partial \mathbf{R}}{\partial \mathbf{y}} \frac{\partial \mathbf{F}}{\partial \mathbf{b}'} \mathbf{S}_{\mathbf{b}} \left(\frac{\partial \mathbf{F}}{\partial \mathbf{b}'} \right)^{\mathrm{T}} \left(\frac{\partial \mathbf{R}}{\partial \mathbf{y}} \right)^{\mathrm{T}}$$

$$S_{\mathbf{b}} = E\{(\mathbf{b}' - \hat{\mathbf{b}})(\mathbf{b}' - \hat{\mathbf{b}})^{\mathrm{T}}\}$$

- Where $\mathbf{S_b}$ is the forward model parameter error covariance matrix
  - very hard to determine

We will assume this term to be small

Note (again) if the satellite instrument changes or if the instrument is not fully characterized this assumption is wrong.

Mean Forward Model Error and Covariance

$$\varepsilon_F = \frac{\partial \mathbf{R}}{\partial \mathbf{y}} \Delta_{\mathbf{F}}(\mathbf{x}, \mathbf{b}, \mathbf{b}')$$

- Remember Δ_F is the difference between the forward function and the forward model
- Modeling error can be hard to evaluate because it requires a model for the forward function which includes the correct physics
- If the correct physics is known and can be modeled accurately by the forward model, then evaluating modeling error is straightforward.

We will assume this term to be small

Note if forward function is not known in detail, or so horrendously complex that no proper model is feasible, this assumption may be wrong.

Note if the instrumentation changes character in orbit, this assumption will be wrong

Mean Retrieval Noise and Covariance

$$\varepsilon_m = \frac{\partial \mathbf{R}}{\partial \mathbf{y}} \varepsilon$$

- Mean Measurement noise is:
  - Assumed random
  - Assumed unbiased
  - Assumed uncorrelated between channels

$$\overline{\varepsilon_m} = \frac{\partial \mathbf{R}}{\partial \mathbf{y}} \varepsilon$$

$$\mathbf{S}_m = \frac{\partial \mathbf{R}}{\partial \mathbf{y}} \mathbf{S}_\varepsilon \left(\frac{\partial \mathbf{R}}{\partial \mathbf{y}} \right)^{\mathrm{T}}$$

$$\mathbf{S}_\varepsilon = E\{\varepsilon \varepsilon^{\mathrm{T}}\}$$

We will NOT assume either term to be small

Retrieval Error Overview

$$\hat{\mathbf{x}} - \mathbf{x} = (\mathbf{A} - \mathbf{I})(\mathbf{x} - \mathbf{x}_a) + \frac{\partial \mathbf{R}}{\partial \mathbf{y}} \frac{\partial \mathbf{F}}{\partial \mathbf{b}'}(\mathbf{b}' - \hat{\mathbf{b}}) + \frac{\partial \mathbf{R}}{\partial \mathbf{y}} \Delta_{\mathbf{F}}(\mathbf{x}, \mathbf{b}, \mathbf{b}') + \frac{\partial \mathbf{R}}{\partial \mathbf{y}} \varepsilon$$

Smoothing Parameters Model Noise

$$\overline{\varepsilon_s} = 0, \mathbf{S}_s \neq 0 \quad \overline{\varepsilon_f} = 0, \mathbf{S}_f = 0 \quad \overline{\varepsilon_F} = 0, \mathbf{S}_F = 0 \quad \overline{\varepsilon_m} \neq 0, \mathbf{S}_m \neq 0$$

- Summing up the **mean Retrieval Error**
  - assumed only contribution by the retrieval noise term):

$$\overline{\varepsilon_{\hat{x}}} = \overline{\varepsilon_s} = \frac{\partial \mathbf{R}}{\partial \mathbf{y}} \varepsilon$$

- Summing up the **Retrieval Error Covariance**
  - assumed only contributions from the smoothing and the retrieval noise terms)

$$\mathbf{S}_{\hat{\mathbf{x}}} = \mathbf{S}_s + \mathbf{S}_m$$

$$\mathbf{S}_{\hat{\mathbf{x}}} = (\mathbf{A} - \mathbf{I})\mathbf{S}_a(\mathbf{A} - \mathbf{I})^{\mathrm{T}} + \frac{\partial \mathbf{R}}{\partial \mathbf{y}} \mathbf{S}_{\varepsilon} \left(\frac{\partial \mathbf{R}}{\partial \mathbf{y}} \right)^{\mathrm{T}}$$

Retrieval Error Overview

- Obviously there are many assumptions regarding the errors from satellite retrievals using Rodger's approach
- This is why careful correlated measurements (using independent instruments like balloons, airborne datasets or high precision accurate satellite profiles) for satellite validations must be performed throughout the life of the satellite to insure that these assumptions remain valid

Retrieval Data Assimilation

$$\mathbf{X}_a = \mathbf{X}_b + \mathbf{W}\left[\hat{\mathbf{x}} - H(\mathbf{X}_b)\right]$$

$$\mathbf{W} = f(\mathbf{S}_{\hat{\mathbf{x}}},\ldots)$$

- In the past the above relationship was used for the data assimilation
  - $\hat{\mathbf{x}}$ the calculated retrieval
  - $H(\mathbf{X}_b)$ the model background converted into the retrieval space using the observation operator which did **NOT** account for the added *a priori* information in the retrieval

New Assimilation of Retrieval Method

In the past: $$\mathbf{X}_a = \mathbf{X}_b + \mathbf{W}\left[\hat{\mathbf{x}} - H(\mathbf{X}_b)\right]$$

Taking into account: $$\hat{\mathbf{x}} = \mathbf{A}\mathbf{x} + (\mathbf{I} - \mathbf{A})\mathbf{x}_a + \varepsilon_{\mathbf{y}}$$

Modify the retrieval $$\hat{\mathbf{x}}' = \hat{\mathbf{x}} + (\mathbf{I} - \mathbf{A})\mathbf{x}_a = \mathbf{A}\mathbf{x} + \varepsilon_{\mathbf{y}}$$

Modify the H-operator $$H'(\mathbf{X}_b) = \mathbf{A}H(\mathbf{X}_b)$$

$$\mathbf{X}_a = \mathbf{X}_b + \mathbf{W}\left[\hat{\mathbf{x}}' - H'(\mathbf{X}_b)\right]$$

Now we have accounted for *a priori* information in the retrieval

New Assimilation of Retrieval Method Error Analysis

Previously: subtract true state vector from both sides

$$\mathbf{X}_a = \mathbf{X}_b + \mathbf{W}\left[\hat{\mathbf{x}} - H(\mathbf{X}_b)\right]$$

$$\hat{\mathbf{x}} - \mathbf{x} = \mathbf{A}\mathbf{x} - \mathbf{x} + (\mathbf{I} - \mathbf{A})\mathbf{x}_a + \varepsilon_{\mathbf{y}}$$

Now: subtract averaging kernel*true state vector from both sides

$$\mathbf{X}_a = \mathbf{X}_b + \mathbf{W}\left[\hat{\mathbf{x}}' - \mathbf{A}H(\mathbf{X}_b)\right]$$

$$\hat{\mathbf{x}}' - \mathbf{A}\mathbf{x} = \mathbf{A}\mathbf{x} - \mathbf{A}\mathbf{x} + \varepsilon_{\mathbf{y}}$$

New Error Analysis

$$\hat{\mathbf{x}}' - \mathbf{A}\mathbf{x} = \varepsilon_{\mathbf{y}}$$

$$\hat{\mathbf{x}}' - \mathbf{A}\mathbf{x} = \frac{\partial \mathbf{R}}{\partial \mathbf{y}} \frac{\partial \mathbf{F}}{\partial \mathbf{b}'}(\mathbf{b}' - \hat{\mathbf{b}}) + \frac{\partial \mathbf{R}}{\partial \mathbf{y}} \Delta_{\mathbf{F}}(\mathbf{x}, \mathbf{b}, \mathbf{b}') + \frac{\partial \mathbf{R}}{\partial \mathbf{y}} \varepsilon$$

Parameters Model Noise

$$\overline{\varepsilon_f} = 0, \mathbf{S}_f = 0 \quad \overline{\varepsilon_F} = 0, \mathbf{S}_F = 0 \quad \overline{\varepsilon_m} \neq 0, \mathbf{S}_m \neq 0$$

No longer a smoothing term in the error analysis but all other terms the same as before so:

$$\overline{\varepsilon_{\hat{x}'}} = \frac{\partial \mathbf{R}}{\partial \mathbf{y}} \varepsilon \qquad \mathbf{S}_{\hat{\mathbf{x}}'} = \frac{\partial \mathbf{R}}{\partial \mathbf{y}} \mathbf{S}_{\varepsilon} \left(\frac{\partial \mathbf{R}}{\partial \mathbf{y}} \right)^{\mathrm{T}}$$

Calculating New Retrieval Error Covariance

- New Retrieval Error Covariance:

$$\mathbf{S}_{\hat{\mathbf{x}}'} = \mathbf{S}_m$$

- Given the old Retrieval Error Covariance:

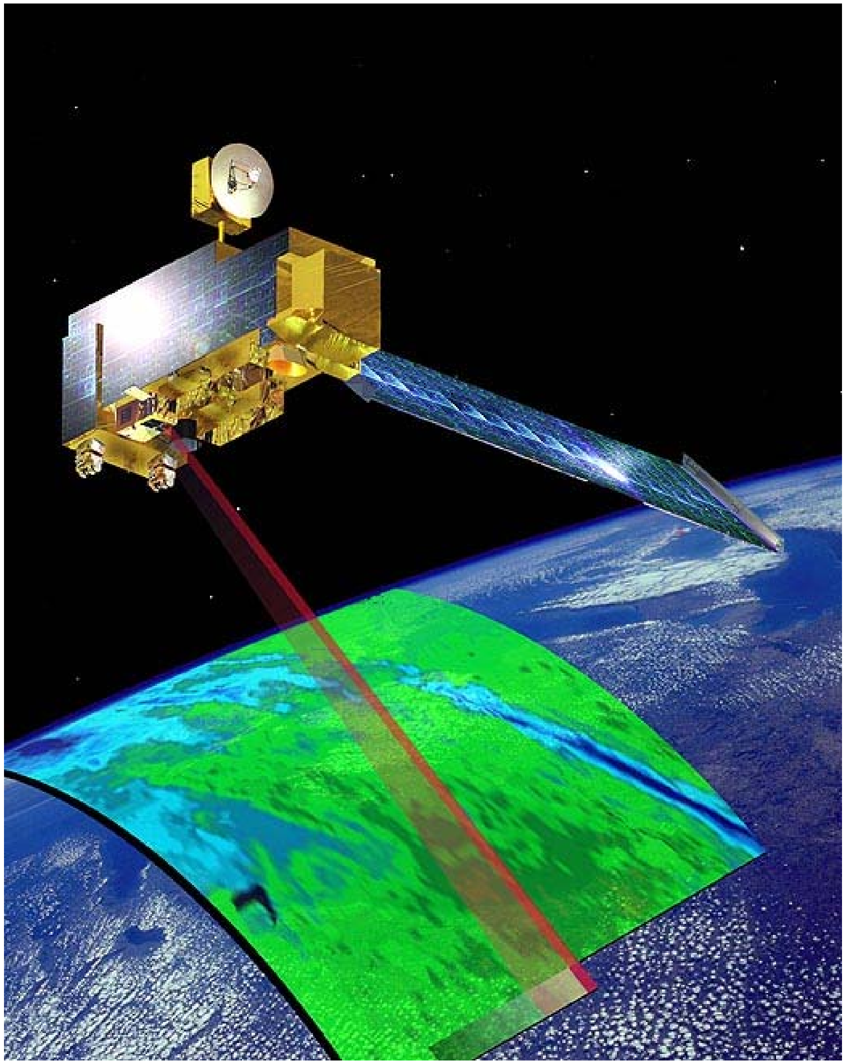
$$\mathbf{S}_{\hat{\mathbf{x}}} = \mathbf{S}_s + \mathbf{S}_m$$

- Combining: $\mathbf{S}_{\hat{\mathbf{x}}'} = \mathbf{S}_{\hat{\mathbf{x}}} - \mathbf{S}_s$

$$\mathbf{S}_{\hat{\mathbf{x}}'} = \mathbf{S}_{\hat{\mathbf{x}}} - (\mathbf{A} - \mathbf{I})\mathbf{S}_a(\mathbf{A} - \mathbf{I})^{\mathrm{T}}$$

- So the New Retrieval Error Covariance is the difference between the Old Retrieval Error Covariance and the contribution of the *a priori* error covariance.

Conclusions



Artist depiction of NASA terra Satellite
with MOPITT instrument
http://www.space.gc.ca/asc/eng/apogee/2005/02_mopitt.asp

- Incorporation of satellite retrievals into data assimilation is not always straight forward. Careful consideration to the quantities assimilated as well as to the errors of these quantities must be taken
- Some of the issues with the assimilation of retrievals in the past have been due to the inclusion of the *a priori* information in these retrievals.
- These issues may be dealt with through removal of *a priori* data from the retrieval within the assimilation system (I only presented one method -- there are others)

Newly Added Slide

Other Assimilation Methods

Shown Method 1:

$$\mathbf{X}_a = \mathbf{X}_b + \mathbf{W}\left[\hat{\mathbf{x}}' - H'(\mathbf{X}_b)\right]$$

$$\hat{\mathbf{x}}' = \hat{\mathbf{x}} - (\mathbf{I} - \mathbf{A})\mathbf{x}_a$$

$$H'(\mathbf{X}_b) = \mathbf{A}H(\mathbf{X}_b)$$

Method 2:

$$\mathbf{X}_a = \mathbf{X}_b + \mathbf{W}\left[\hat{\mathbf{x}}'' - H''(\mathbf{X}_b)\right]$$

$$\hat{\mathbf{x}}'' = \hat{\mathbf{x}} - \mathbf{x}_a$$

$$H''(\mathbf{X}_b) = \mathbf{A}\left(H(\mathbf{X}_b) - \mathbf{x}_a\right)$$

Method 3:

$$\mathbf{X}_a = \mathbf{X}_b + \mathbf{W}\left[\hat{\mathbf{x}} - H'''(\mathbf{X}_b)\right]$$

$$H'''(\mathbf{X}_b) = \mathbf{A}H(\mathbf{X}_b) + (\mathbf{I} - \mathbf{A})\mathbf{x}_a$$

Thank you!